Nonlinear dynamics induced anomalous Hall effect in Topological Insulators

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G.-L. Wang, H.-Y. Xu, and Y.-C. Lai. "Nonlinear dynamics induced anomalous Hall effect in topological insulators." *Scientific reports* **6**, 19803 (2016).

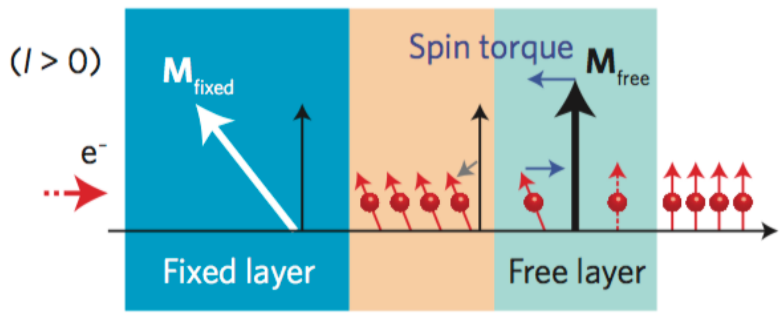
Background

Spintronics

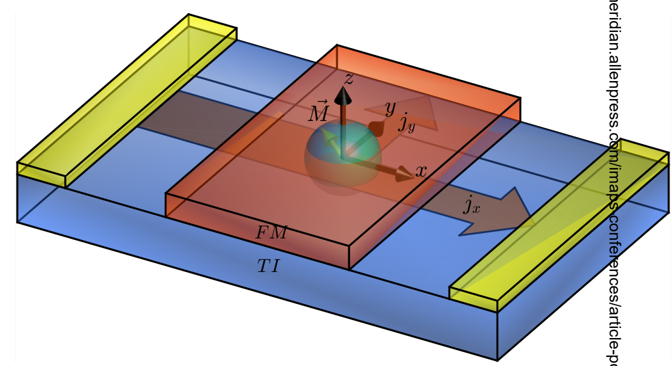
Current

Spin-transfer Torque

Magnetization



Magnetic tunneling junction (MTJ)



Topological Insulator- Ferromagnet

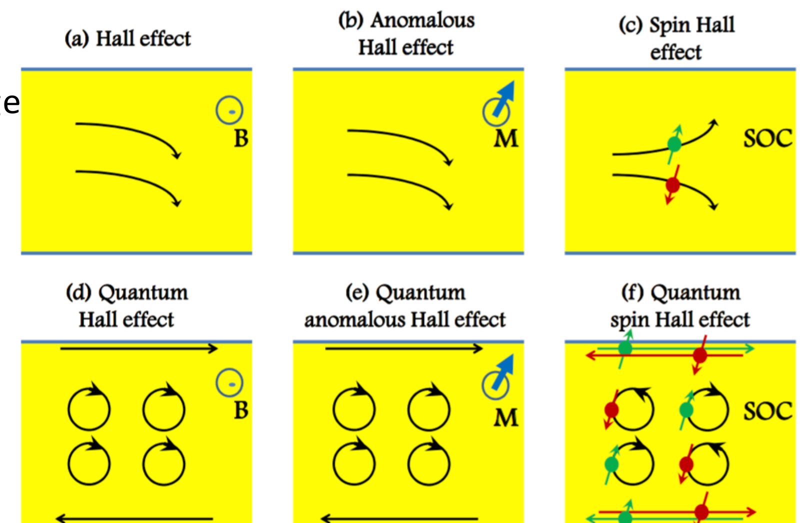
Hall Effect

Current induces transverse voltage

Current

Lorentz;
Landau Level;
SOC,...

Transverse voltage

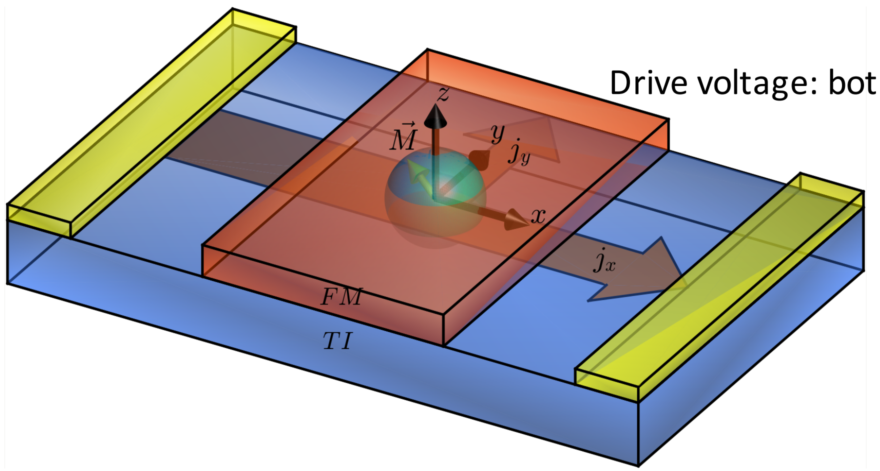


Spintronics: N. Locatelli, *et. al.*, “Spin-torque building blocks.” Nature materials 13.1 (2014): 11-20.

Hall Effect: C.-X. Liu, *et. al.*, “The quantum anomalous Hall effect”, arXiv:1508.07106, (2015)

A.R. Mellnik *et. al.*, “Spin-transfer torque generated by a topological insulator”, Nature. 2014 Jul 24;**511**(7510):449-51

System Configuration



Drive voltage: both ac and dc

FM: Ferromagnet ($Ni_{81}Fe_{19}^{*}$~10nm)

TI: Topological Insulator ($Bi_2Se_3^{*}$~8nm)

$$n_x^2 + n_y^2 + n_z^2 = 1$$

Ferromagnet: $$\dot{\mathbf{n}} = -\frac{D}{\hbar}\mathbf{n} \times \hat{x} + \alpha_G \mathbf{n} \times \dot{\mathbf{n}} + \frac{1}{\hbar}\mathbf{T}, \qquad \boldsymbol{T} = \langle \boldsymbol{\sigma} \rangle \times \boldsymbol{m} = \xi \langle \boldsymbol{\sigma} \rangle \times \boldsymbol{n}$$

1st term: procession, $D \sim$ aniostropy energy; 2nd term: damping, $\alpha_G \sim$ Gilbert damping constant; 3rd term: spin-transfer torque, (torque from the spin-polarized current)

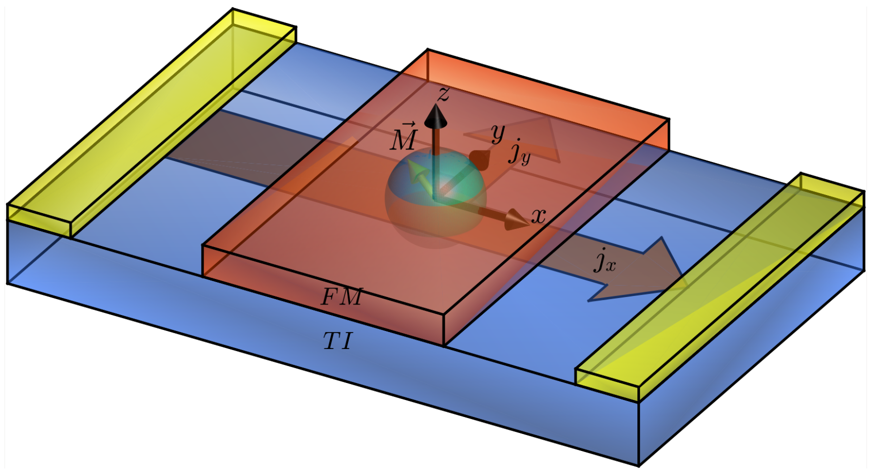
Topological insulator: $$H = \hbar v_F(\boldsymbol{\sigma} \times \boldsymbol{k}) \cdot \hat{z} + \boldsymbol{m} \cdot \boldsymbol{\sigma}\Theta(x)\Theta(L - x),$$

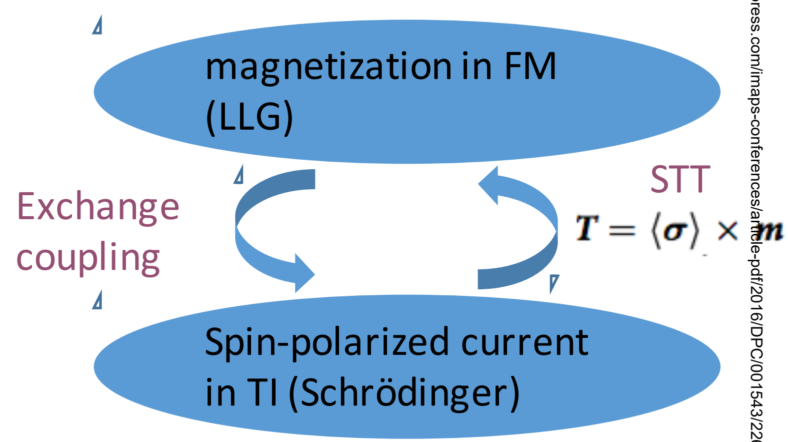
1st term: ideal TI, $\sigma \sim$ Pauli matrices; 2nd term: exchange interaction, $m \sim$ magnetization in FM

*A.R. Mellnik *et. al.*, “Spin-transfer torque generated by a topological insulator”, Nature. 2014 Jul 24;**511**(7510):449-51

Simulation

$$\dot{\mathbf{n}} = -\frac{D}{\hbar}\mathbf{n}\times\hat{x} + \alpha_G \mathbf{n}\times\dot{\mathbf{n}} + \frac{1}{\hbar}\mathbf{T},$$





$$H = \hbar v_F(\boldsymbol{\sigma}\times\boldsymbol{k})\cdot\hat{z} + \boldsymbol{m}\cdot\boldsymbol{\sigma}\Theta(x)\Theta(L-x),$$

Channel Current $$j_x = -\frac{k_F}{2\pi}\frac{e^2}{\hbar}V\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta\,|t|^2\cos\theta,$$

Hall Current $$j_y = -\frac{k_F}{2\pi}\frac{e^2}{\hbar}V\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta\,|t|^2\sin\theta,$$

$$t = \frac{-4\cos\theta\hbar v_F\tilde{k}_x}{\alpha(A + ie^{i\theta}B)}$$

$$\langle\boldsymbol{\sigma}\rangle = -\frac{1}{ev_F}\boldsymbol{j}\times\hat{\boldsymbol{z}}$$

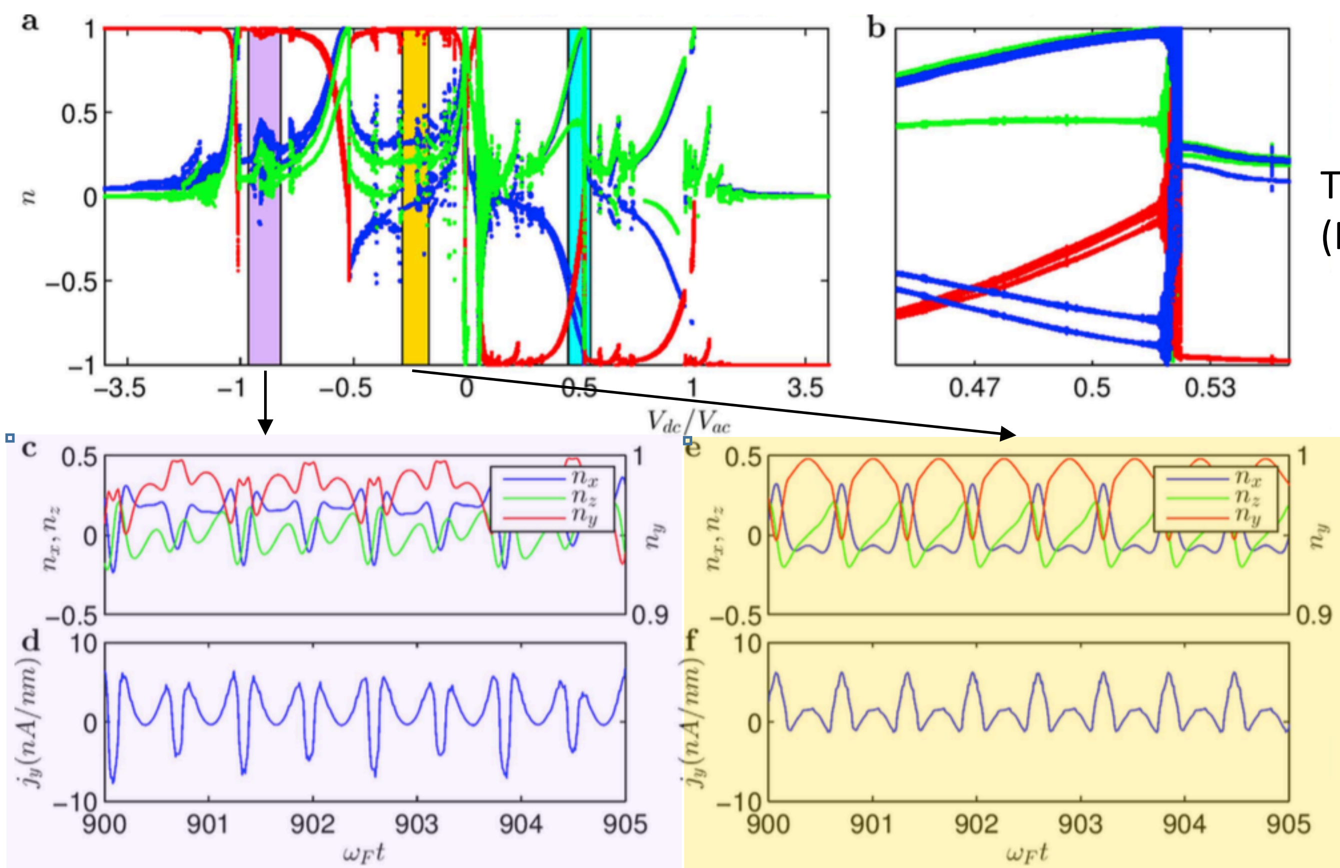
t ~ transmission coefficient in TI
$|t|^2$ is even function at no FM
FM makes $|t|^2$ not even function

j_y ~ anomalous Hall current*

T. Yokoyama, "Current-induced magnetization reversal on the surface of a topological insulator." *Phys. Rev. B* **84,** 113407 (2011).
*Y. G. Semenov, *et. al.*, "Voltage-driven magnetic bifurcations in nanomagnet-topological insulator heterostructures." *Phys. Rev. B* **89,** 201405 (2014).

Bifurcation

Overview:
Initial: (1,0,0)
Blue: n_x
Red: n_y
Green: n_z



Transition Region
(Multistability)

Magnetization:
(FM)

Current:
(TI)

Chaotic Region

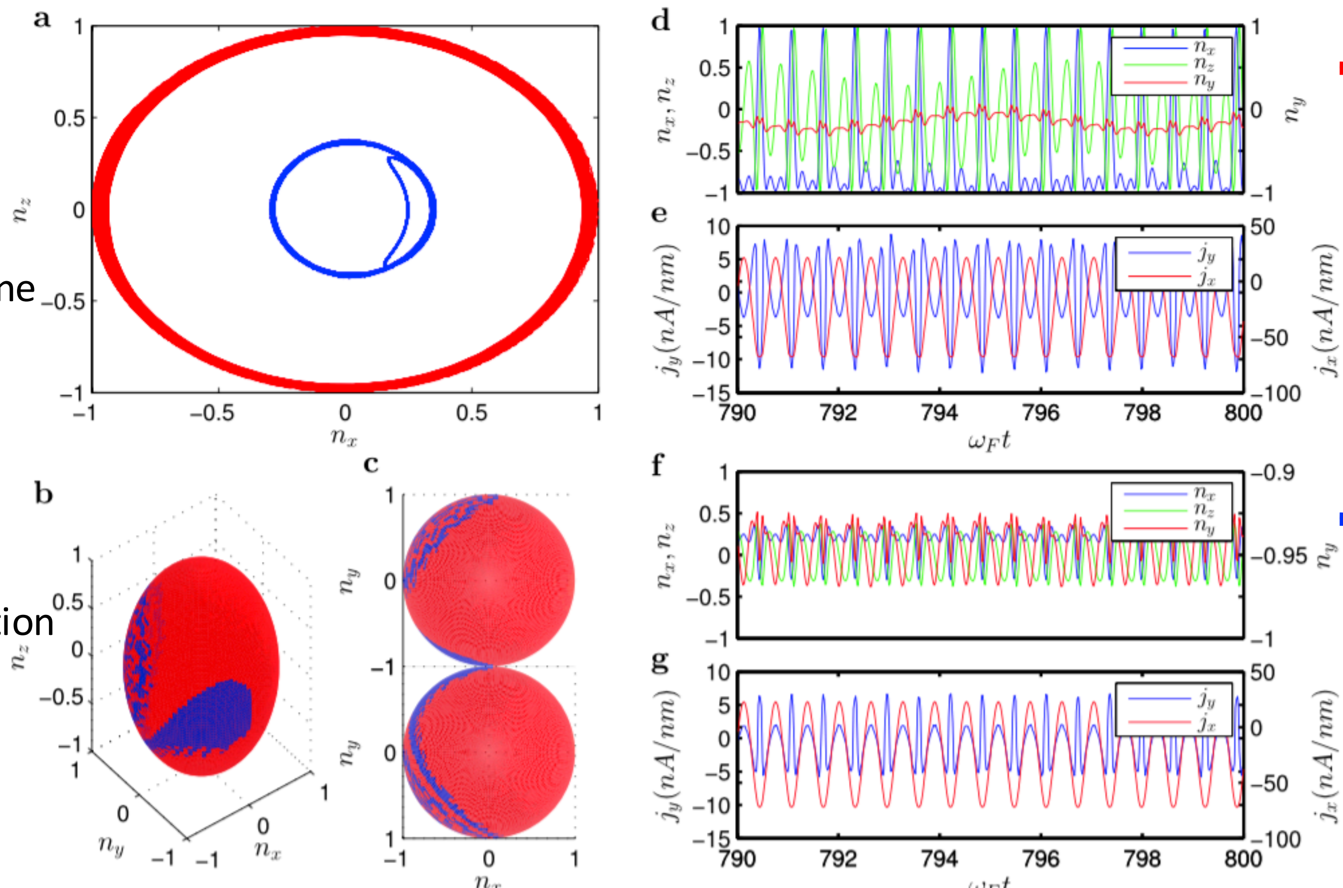
Phase Synchronization

Multistability: Sample

$\frac{V_{dc}}{V_{ac}} = 0.5179$

Final state trajectories: $n_x - n_z$ plane

Initial Condition & Final State



Red State:
n_y is small
$n_y \sim 0$

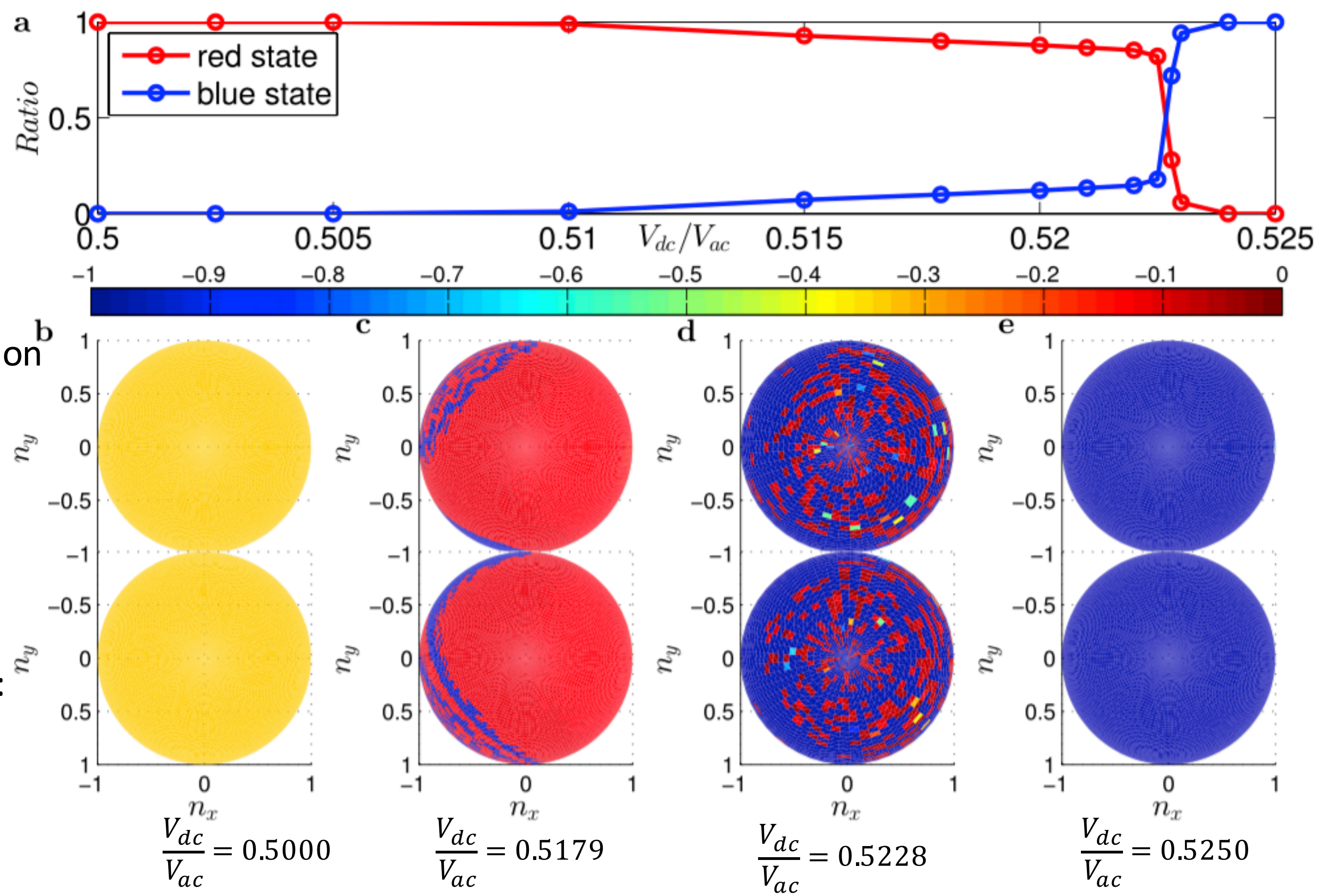
j_y (blue curve) change phase

Blue State:
n_y is large
$n_y \sim -1$

$n_x^2 + n_y^2 + n_z^2 = 1$, initial condition is a sphere.

Q: Can we control the final state?

Multistability: Transition

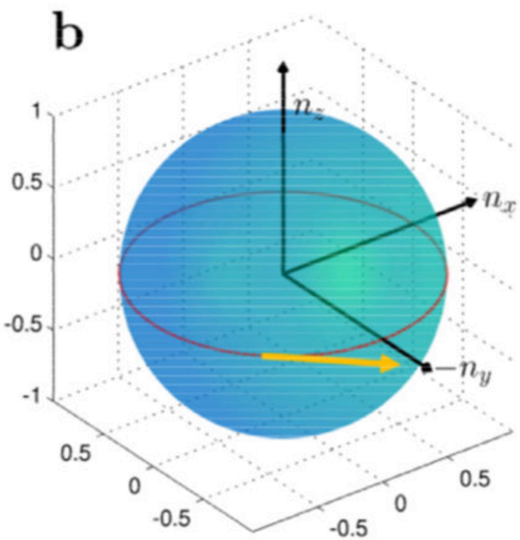


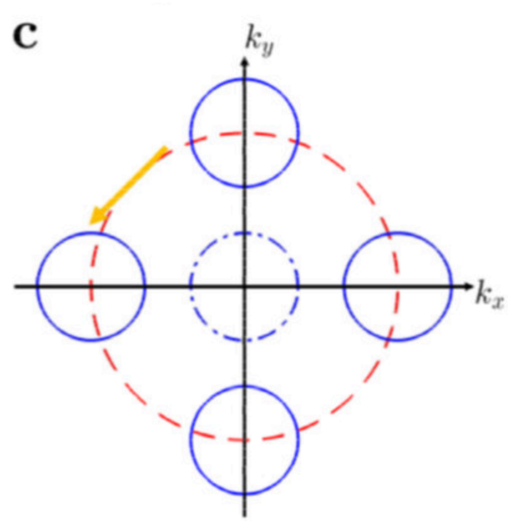
Initial Condition & Final State

Aerial view:

Ground view:

Anomalous Hall Effect





Magnetization in FM changes the Fermi surface of electrons in the wave vector space.

Net j_y induced by m_x without external magnetic field.

Hamiltonian in TI:

$$H = \hbar v_F (\boldsymbol{\sigma} \times \boldsymbol{k}) \cdot \hat{z} + \boldsymbol{m} \cdot \boldsymbol{\sigma}$$

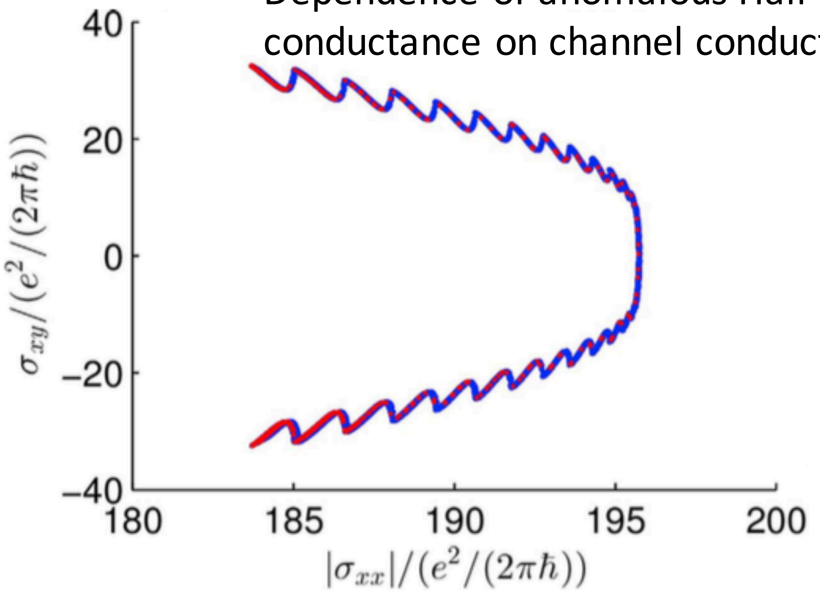
$$H = \hbar v_F \left[\sigma_x \left(k_y + \frac{m_x}{\hbar v_F} \right) - \sigma_y \left(k_x - \frac{m_y}{\hbar v_F} \right) \right] + m_z \sigma_z$$

$$E^2 = \left(k_y + \frac{m_x}{\hbar v_F} \right)^2 + \left(k_x - \frac{m_y}{\hbar v_F} \right)^2$$

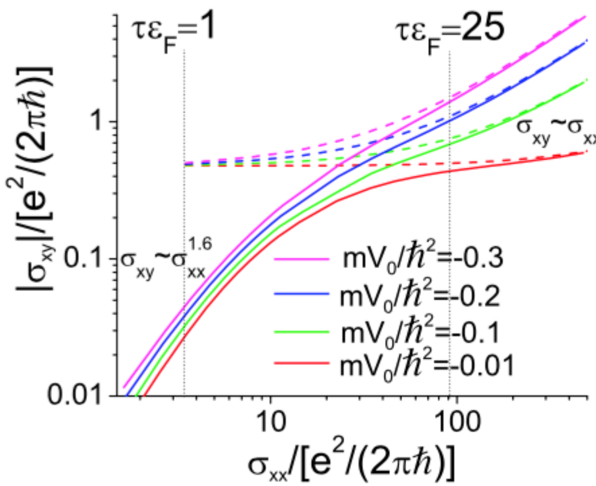
m_x shifts k_y effectively. Contribute to a net current along y direction.

Anomalous Hall Current

Dependence of anomalous Hall conductance on channel conductance.



Conventional:



*Nagaosa, N., Sinova, J., Onoda, S., MacDonald, A. H. & Ong, N. P. Anomalous hall e ect. *Rev. Mod. Phys.* **82,** 1539–1592 (2010).

$$\sigma_{xx} = \frac{j_x}{V/L} = -k_F L \frac{e^2}{2\pi\hbar}\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta\, |t|^2 \cos\theta,$$

$$\sigma_{xy} = \frac{j_y}{V/L} = -k_F L \frac{e^2}{2\pi\hbar}\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta\, |t|^2 \sin\theta.$$

$k_F L$ ~ Number of available carriers;

$\frac{e^2}{2\pi\hbar}$ ~ Fundamental conductance unit;

Summary

We study the exchange coupling between magnetization in FM and the spin-polarized current in TI and discover rich nonlinear dynamical behavior.

1. We find chaotic regime, phase synchronization regime and multistability regime.
2. The multistability regime provides an efficient way to control the state of the magnetization in the FM and the phase of the anomalous Hall current in the TI.
3. The system can dynamically induce an anomalous Hall conductance which possesses an unconventional dependence on the channel conductance.

Thank you!