Nonlinear Observability Analysis of Micro-machined Electrostatic Actuators Using Self-Sensing

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Agenda

- 1. Background
- 2. Observability Analysis
- 3. Verifications
- 4. Conclusions

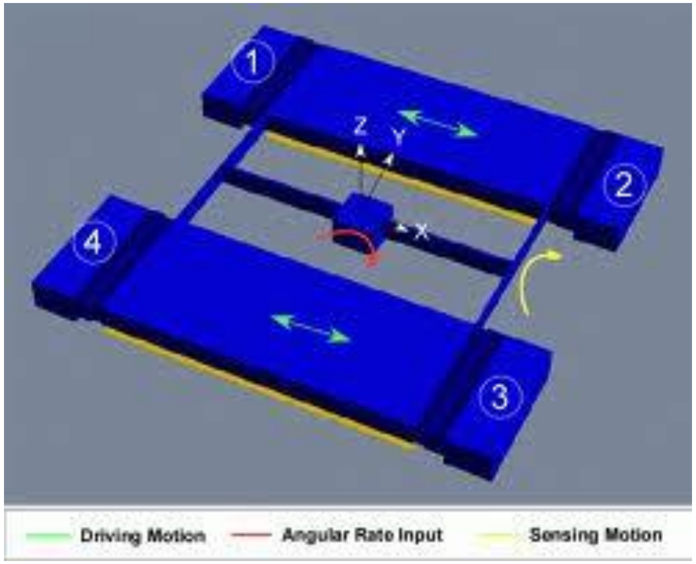
1. Background

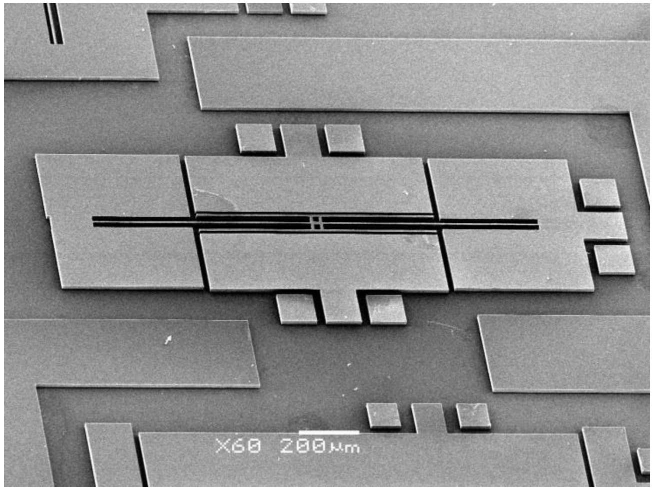
Model of PPAs

Problem Statement

Applications of PPAs

- Electrostatic parallel-plate actuators (PPAs) are used in many types of microelectromechanical systems (MEMS) devices, such as accelerometers , variable capacitors, RF devices and micromirrors.

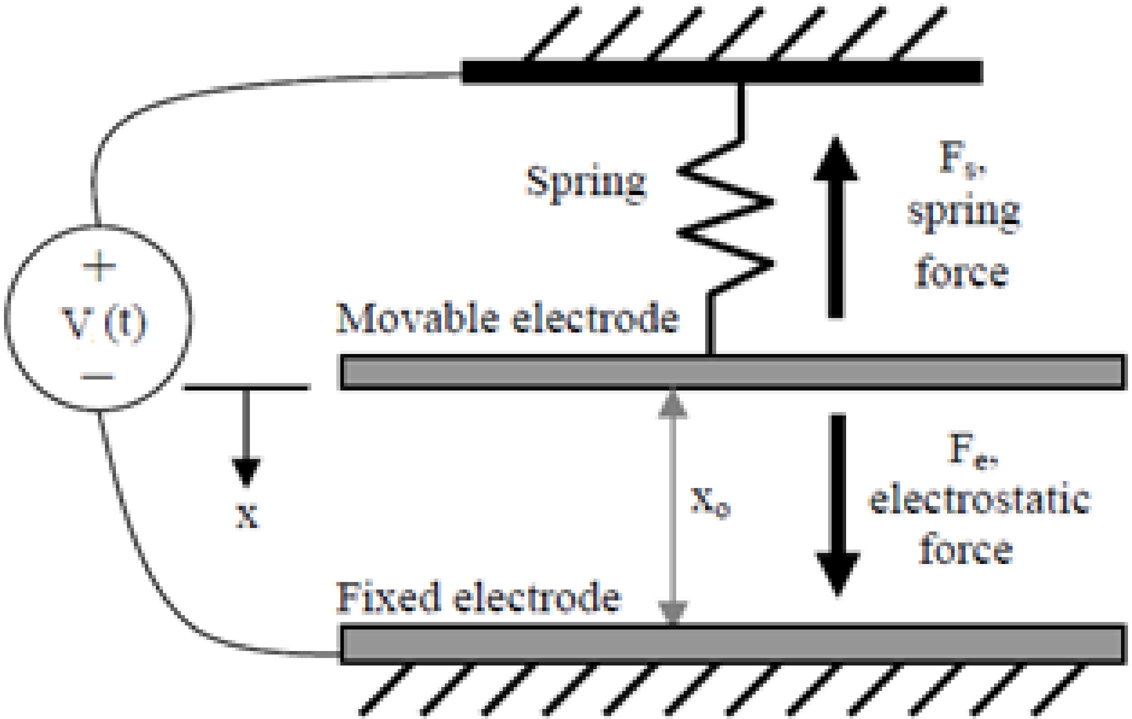




Parallel Plate Actuators

Ignoring gravity and damping force, at equilibrium condition:

Spring force = electrostatic force

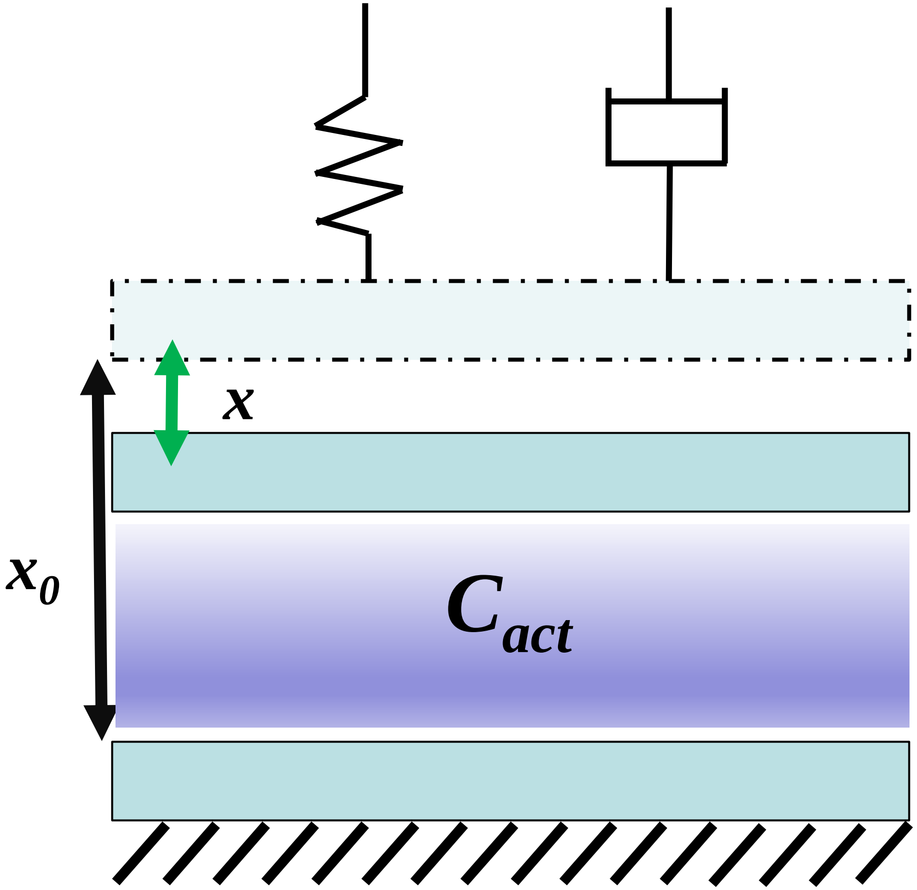


System Dynamics' Equation (1)

$$m\ddot{x} + c\dot{x} + kx = f(x,V) = \frac{\varepsilon A V^2}{2(x_o - x)^2}$$

- A: areas of two electrodes
- m: mass
- c: damping coefficient
- k: spring constant
- x_0: initial gap
- $\varepsilon_0\varepsilon_r$: permittivity ≈ 8.854 pf/m

System Dynamics' Equation (2)



The capacitance between the electrodes is:

$$C_{act} = \frac{\varepsilon_0 \varepsilon_r A}{x_0 - x}$$

Its resonant frequency is:

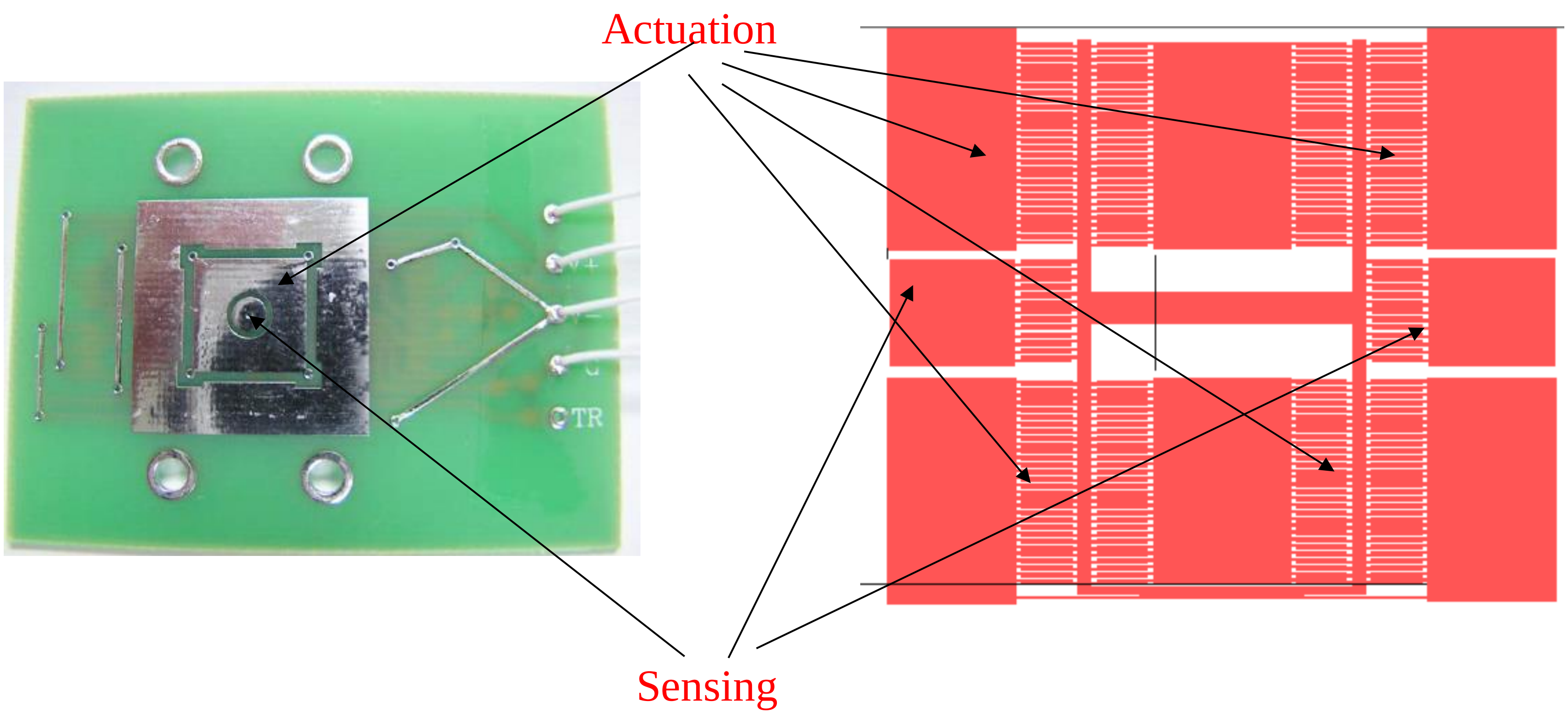
$$\omega_0 = \sqrt{\frac{k}{m}}$$

Main Applications' Issue

- Limited stable range (less than 1/3)
- Typically open loop control just for on/off applications
- Feedback control techniques are required in some advanced applications

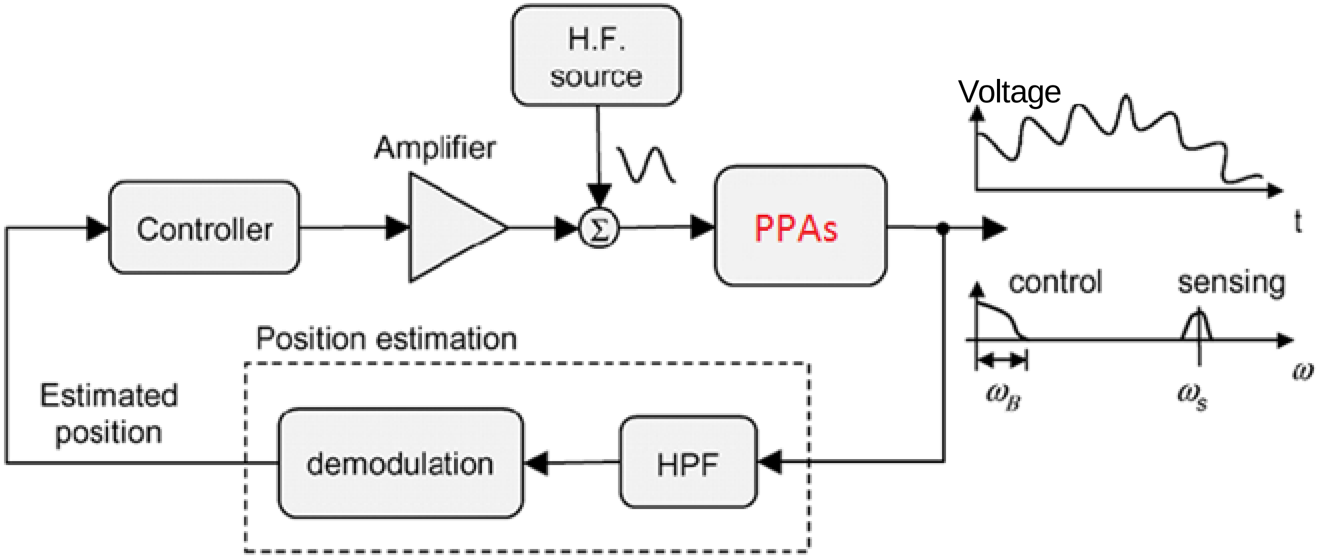
External Sensing for PPAs (1)

- Additional sensing structures



External Sensing for PPAs (2)

- Additional Excitation Source

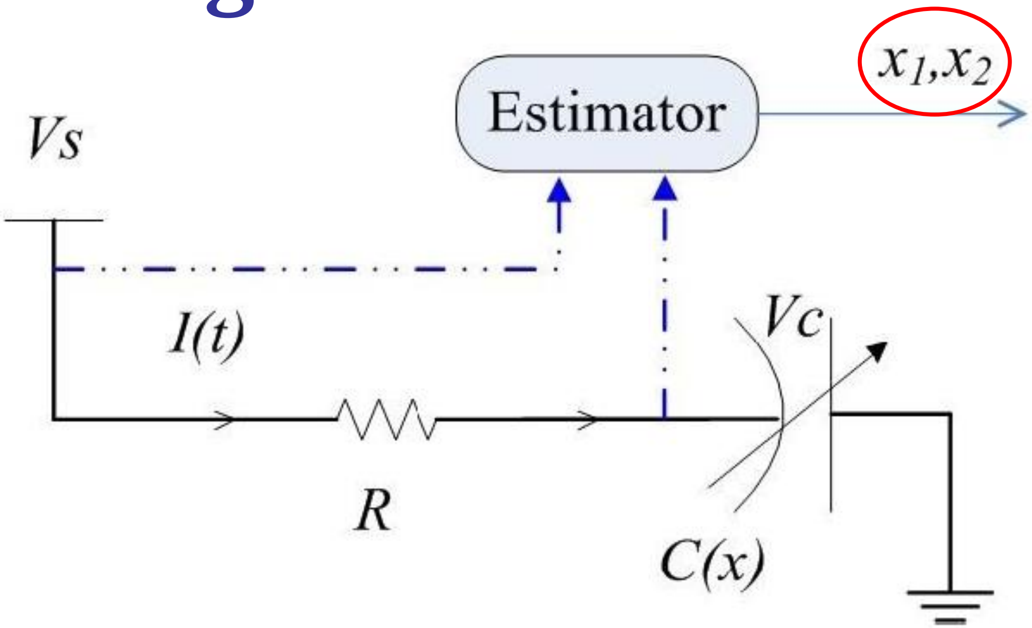


- It needs more electronics.
- May introduce harmonics.

2. Observability Analysis

- Nonlinear Observability Analysis
- Linearized Observability Analysis
- Observability Enhancement

System Configuration

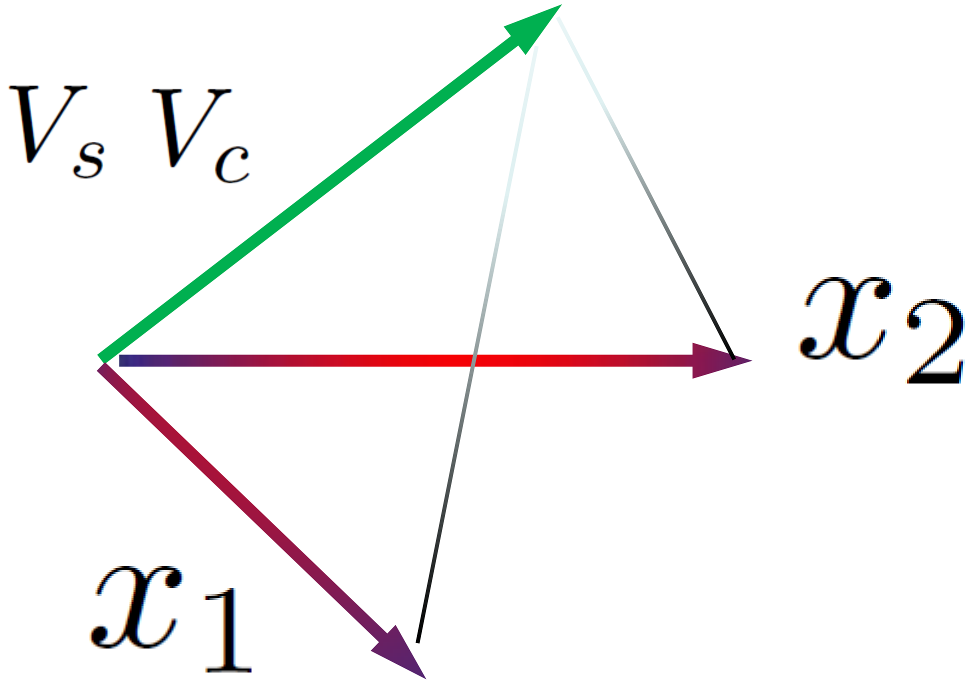


$$m\dot{x}_2 = -cx_2 - kx_1 + \frac{\varepsilon_0\varepsilon_r A V_c^2}{2(x_0 - x_1)^2}$$

$$\dot{V}_c = \frac{1}{c(x_1)}\left(-\frac{V_c}{R} - \frac{V_c\varepsilon_0\varepsilon_r A}{2(x_0 - x_1)^2}x_2 + \frac{V_s}{R}\right)$$

Can we really recovery the information?

- Can we determine the position and velocity by measuring *Vs* and *Vc*?
- Is it observable?



Nonlinear Observability Criteria

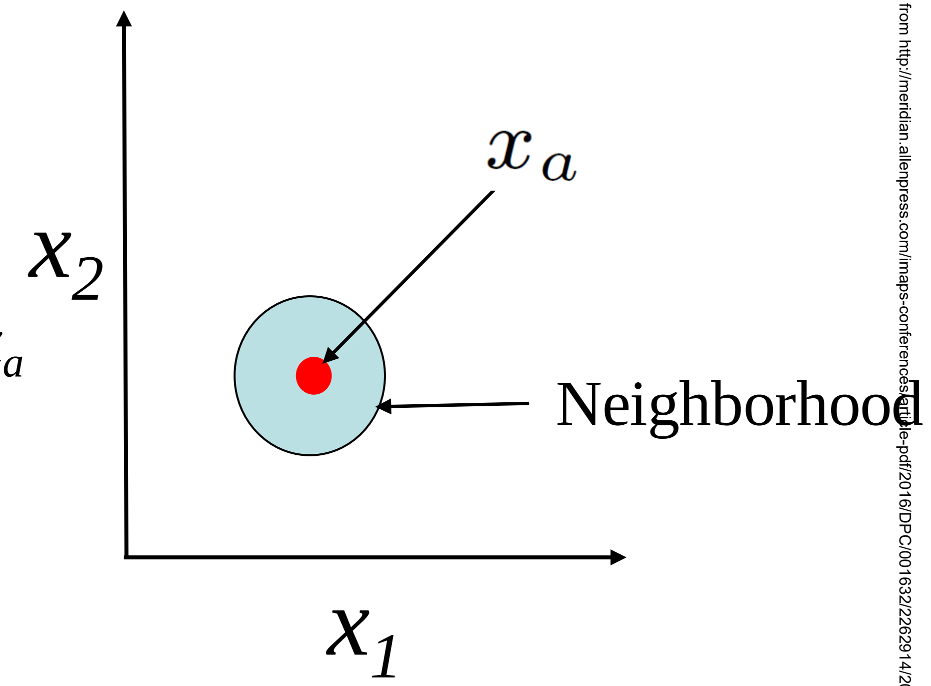
- The given nonlinear system

$$\dot{x} = f(x,u)$$
$$y = h(x).$$

is locally observable around x_a

if On has full rank

$$O_n = \left.\frac{\partial l(x)}{\partial x}\right|_{x=x_a}$$



where $l(x) = \begin{bmatrix} \mathcal{L}_f^0 h \\ \mathcal{L}_f^1 h \\ \mathcal{L}_f^2 h \\ \cdots \\ \mathcal{L}_f^{n-1} h. \end{bmatrix}$ and $\mathcal{L}_f h$ is the Lie derivatie

Determining the Observability (1)

- Calculating O_n for the System:

$$O_n = \left.\frac{\partial l(x)}{\partial x}\right|_{x=x_a} = \begin{bmatrix} 0 & 0 & 1 \\ O_{n21} & O_{n22} & O_{n23} \\ O_{n31} & O_{n32} & O_{n33} \end{bmatrix}$$

$$O_{n21} = \frac{V_c - V_s}{R\varepsilon_0\varepsilon_r A} - \frac{V_c}{2(x_0 - x_1)^2}x_2$$

$$O_{n22} = -\frac{V_c}{2(x_0 - x_1)}$$

$$O_{n23} = -\frac{(x_0 - x_1)}{R\varepsilon_0\varepsilon_r A} - \frac{1}{2(x_0 - x_1)}x_2$$

Determining the Observability (2)

$$O_{n31} = \frac{1}{4A^2(\varepsilon_0\varepsilon_r)^2 mR^2(x_0 - x_1)^4}$$
$$(-3A^3(\varepsilon_0\varepsilon_r)^3 R^2 V_c^3 - 8m(V_c - V_s)(x_0 - x_1)^5$$
$$+ (2A^2(\varepsilon_0\varepsilon_r)^2 R^2 V_c x_0 - x_1)$$
$$(kx_0(x_0 - x_1) - x_2(c(-x_0 + x_1) + mx_2)))$$

$$O_{n32} = \frac{m(4V_c - 3V_s)(x_0 - x_1)^2}{2A\varepsilon_0\varepsilon_r mR(x_0 - x_1)^2} + \frac{A\varepsilon_0\varepsilon_r RV_c(c(x_0 - x_1) - mx_2)}{2A\varepsilon_0\varepsilon_r mR(x_0 - x_1)^2}$$

- Hard to tell the rank by calculating its determinant:

$$|O_n| = O_{21}O_{n32} - O_{n22}O_{n31},$$

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Linear Observability (1)

Linearize the system around a equilibrium point $(x_{1e}, 0, V_{ce})$ and $V_{ce} = V_s$

$$\delta\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -k/m + \frac{\varepsilon_0 \varepsilon_r D V_{ce}^2}{m(x_0 - x_{1e})^3} & -c/m & \frac{\varepsilon_0 \varepsilon_r D V_{ce}}{m(x_0 - x_{1e})^2} \\ 0 & -\frac{1}{2(x_0 - x_{1e})} V_{ce} & -\frac{x_0 - x_{1e}}{R \varepsilon_0 \varepsilon_r D} \end{bmatrix} \delta x$$

$$\delta\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \delta x = A \delta x$$

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Linear Observability (2)

- The system is observable if and only if the following matrix has full rank

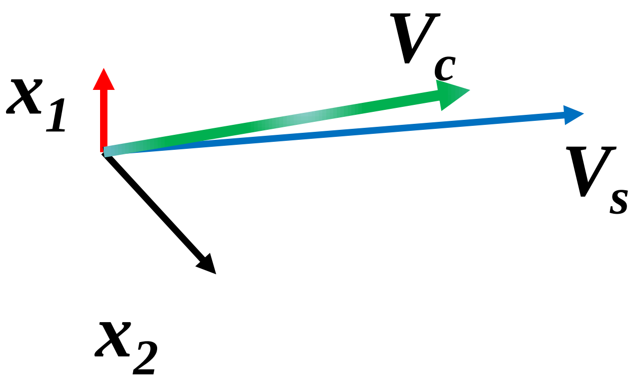
$$Q_o = \begin{bmatrix} 0 & 0 & 1 \\ 0 & a_{32} & a_{33} \\ a_{21}a_{32} & a_{22}a_{32} + a_{32}a_{33} & a_{33}^2 + a_{23}a_{32} \end{bmatrix}$$

- It has full rank if and only if $|Q_o| = a_{21}a_{32}^2$ is not zero.
- The system is observable unless $x_1 = x_0/3$

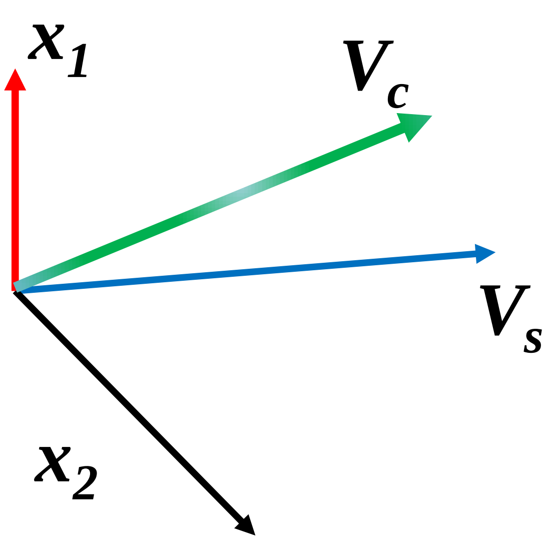
Observable Quality (1)

- Though it's observable, the quality of the observability is uncertain.
- Can we make a reliable PPA to fulfill this technique?

Bad Observability



Good Observability



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Observable Quality (2)

- It's observable if and only if *Qo* has full rank, but the quality of the observability is uncertain.

$$Q_o = \begin{bmatrix} 0 & 0 & 1 \\ 0 & a_{32} & a_{33} \\ a_{21}a_{32} & a_{22}a_{32} + a_{32}a_{33} & a_{33}^2 + a_{23}a_{32} \end{bmatrix}$$

- An example of a matrix that has full rank, but is weakly full rank:

$$\begin{bmatrix} 1 & 1 \\ 1 & 0.99 \end{bmatrix}$$

Observability Enhancement (1)

- Using singular value decomposition (SVD) to analysis Q_o.

$$Q_o = U \Sigma V^T$$

- where $\Sigma = diag(\sigma_1, \sigma_2, \sigma_3)$ is a diagonal matrix, and $\sigma_1 \geq \sigma_2 \geq \sigma_3$ are the singular values of Qo; U and V are orthogonal matrices.
  - Define the observability index as:

$$O_2 = \frac{\sigma_1}{\sigma_3}$$

  - The observability is better when O_2 becomes smaller.

Observability Enhancement (2)

- Directly solving O_2 is difficult.
- The upper bound of O_2 is:

$$\frac{\|Q_o\|_F^3}{|Q_o|} \geq \frac{\sigma_1}{\sigma_3}$$

  - where $\|Q_o\|_F$ is the Frobenius norm of Q_o and

$$|Q_o| = \sigma_1 \sigma_2 \sigma_3$$

Thus, if $|Q_o| = a_{21} a_{32}^2$ is increasing and $\|Q_o\|_F$ is decreasing, the observability O_2 will decrease, which means that actuator's observability is improved (less sensitive to noise and finite word length).

Observability Enhancement (3)

- This observability can be improved if an element in G changes which satisfies:

$$\frac{\partial \left\|Q_o\right\|_F^3}{\partial a_{ij}} \leq \frac{\partial \left|Q_o\right|}{\partial a_{ij}}$$

- Larger series resistance, smaller initial gap and smaller spring constant will improve the observability.

3. Validation

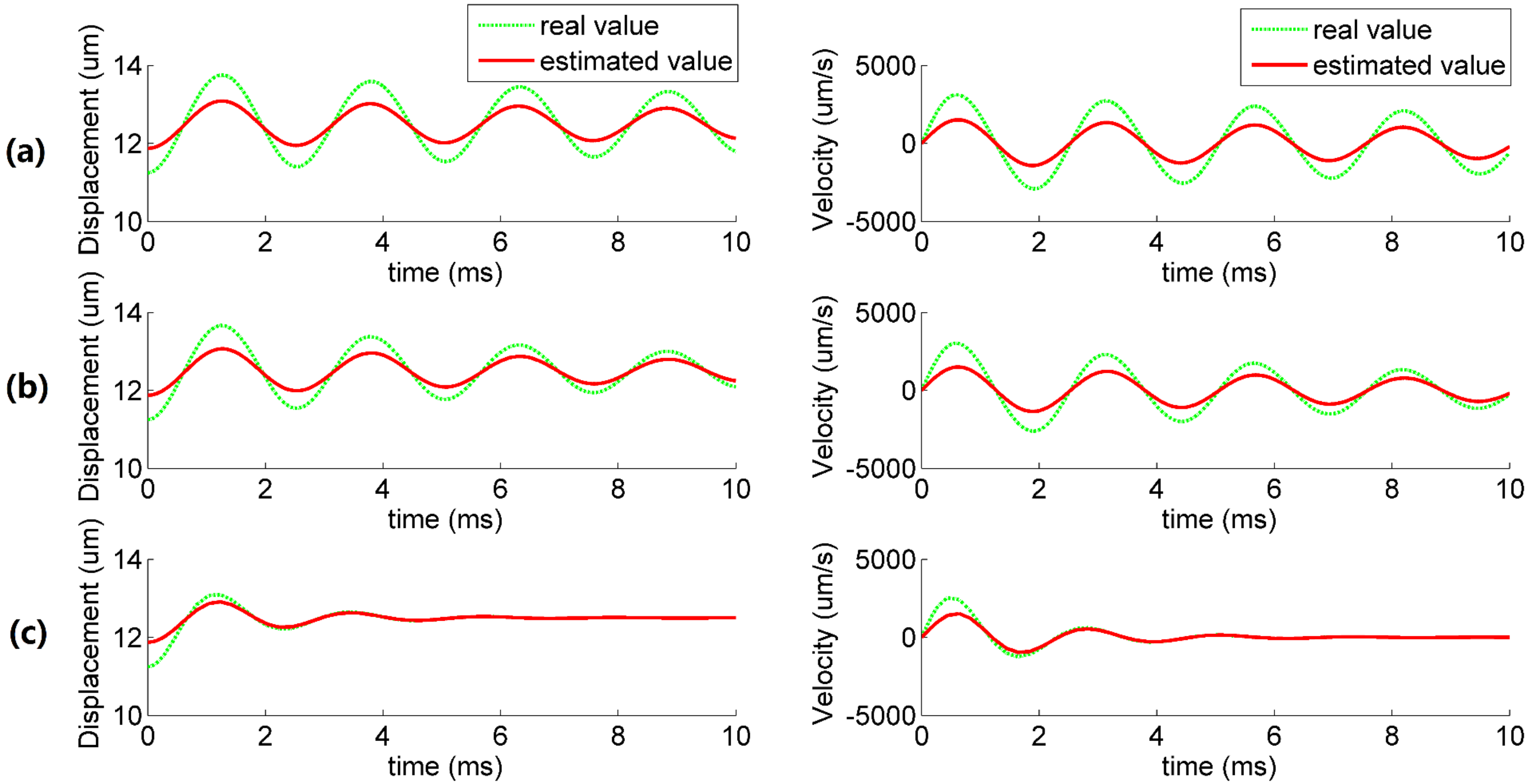
- Feasibility of Self-Sensing
- Performances Using Different Parameters

System Parameters

- A simulation study was performed using the given parameters

Symbol	Description	Value	Unit
k	spring constant	150	N/m
c	damping coefficient	7.5×10^{-4}	$N \cdot s/m$
m	proof mass	7.5	μg
A	overlap area of the electrodes	2.9×2.9	mm
R	series resistance	100 M	Ω
x_0	initial gap distance	10	μm
ε_0	permittivity of free space	8.854×10^{-12}	F/m
ε_r	permittivity of air	1.0006	N/A

Simulation Results (1)



(a) R = 1M Ω, (b) R = 10M Ω and (c) R = 100M Ω

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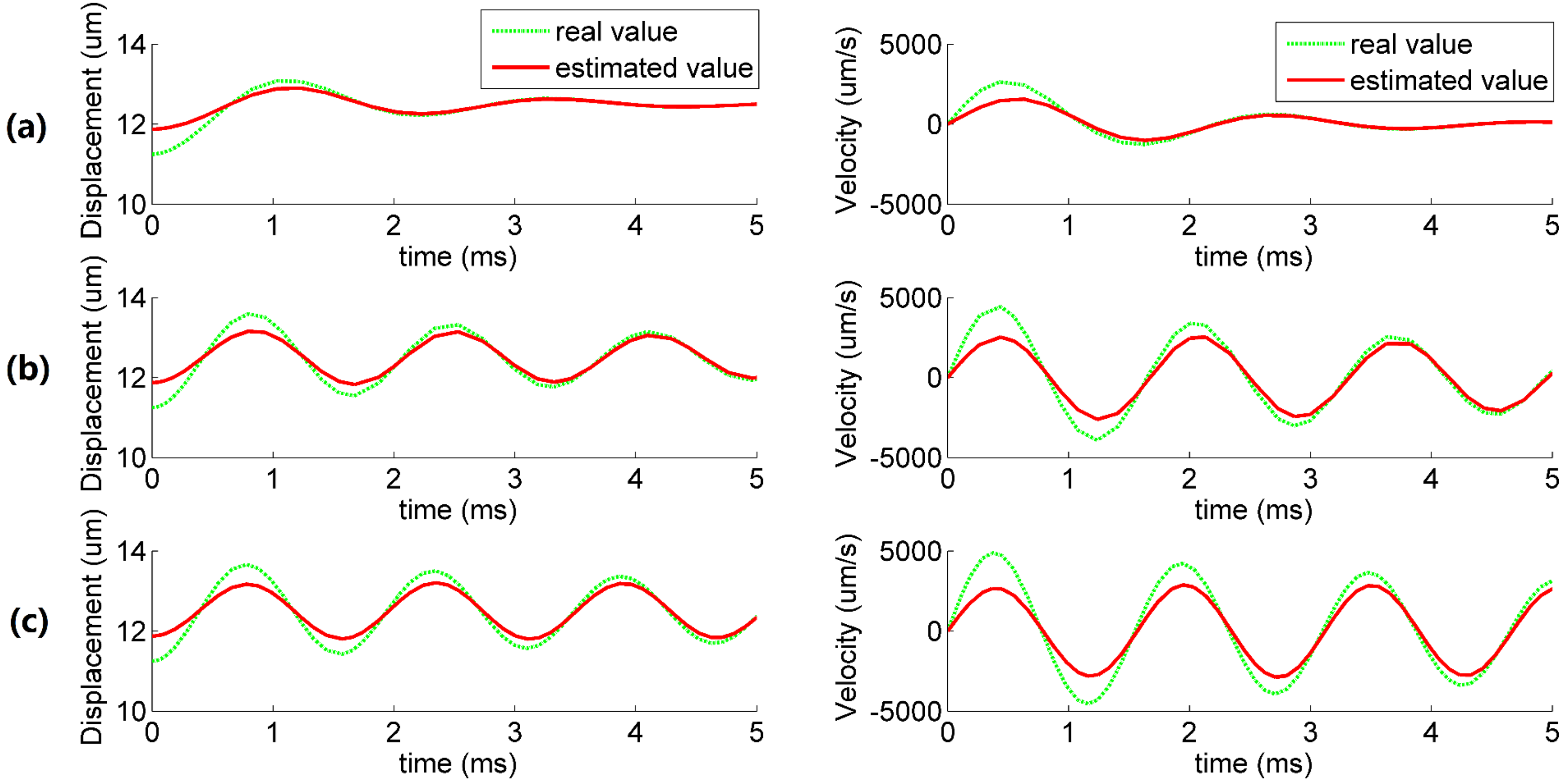
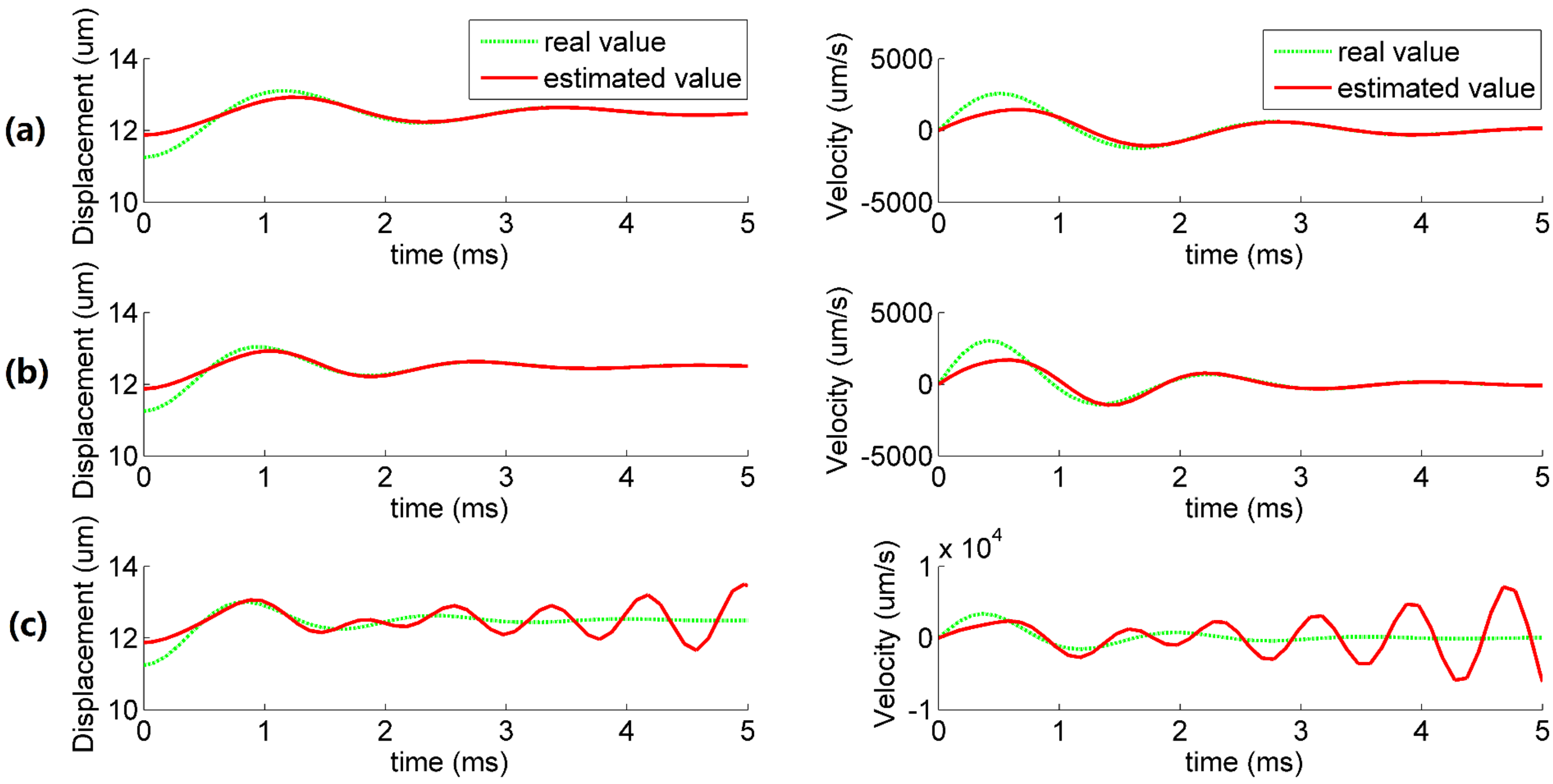
Simulation Results (2)



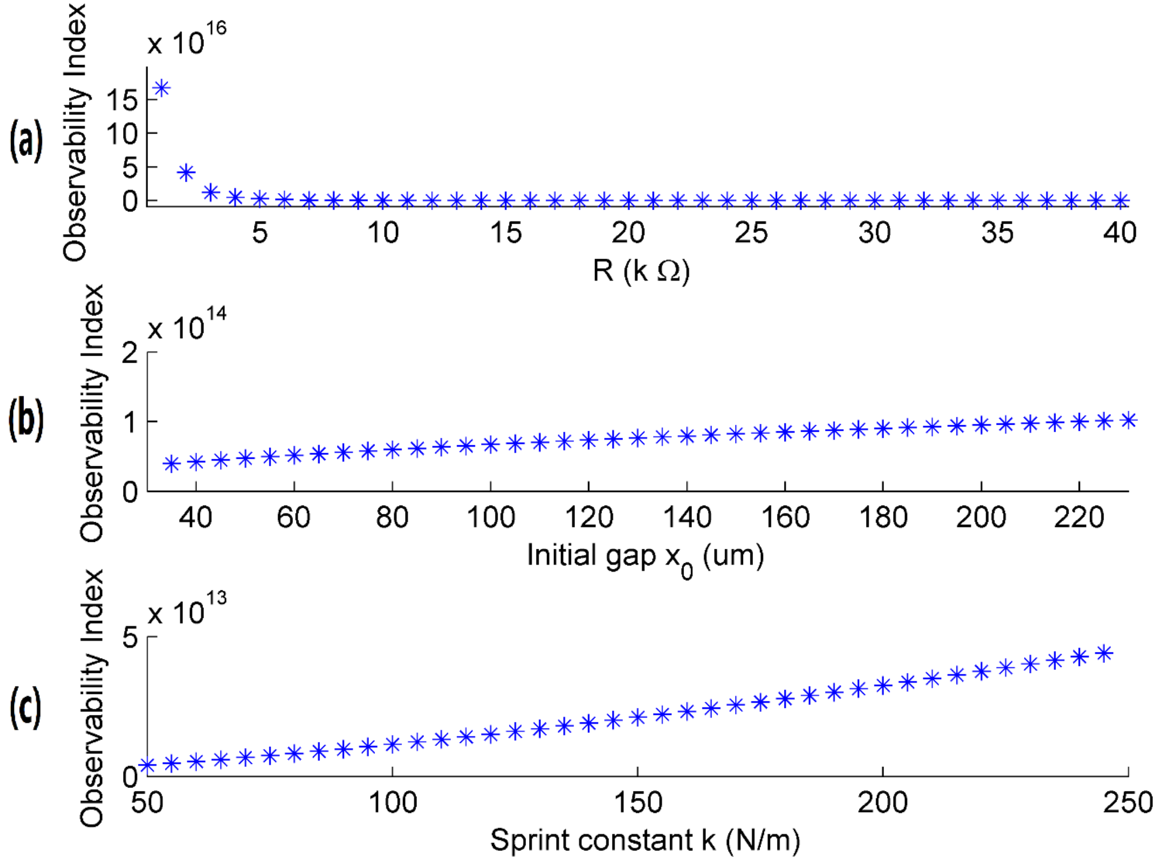
(a) x_0 =50 μm, (b) x_0 =100 μm and (c) x_0 =150 μm

Simulation Results (3)



(a) k =150 N/m, (b) k =200 N/m and (c) k =250 N/m

Simulation Results (4)



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4. Conclusions

- The self-sensing technique to estimate the mechanical states of a class of MEMS electrostatic actuators is feasible when a reasonably soft spring exists and the displacement is nonzero.
- Variation of the actuators' parameters also affect the observability. This technique is improved with a larger series resistance and a smaller initial gap.

Thank You for Your Attention.

Questions?