

## Recent Advances in Practical Metamaterial Engineering

**Khalid Z. Rajab, Yifeng Fan and Yang Hao**

Queen Mary University of London  
School of Electronic Engineering & Computer Science  
London E1 4NS, United Kingdom  
[khalid.rajab@eeecs.qmul.ac.uk](mailto:khalid.rajab@eeecs.qmul.ac.uk)

### Abstract

Metamaterials have been used to implement materials with near-zero and negative values of the relative electric permittivity ( $\epsilon$ ), as well as magnetic permeability ( $\mu$ ). These properties enable potentially significant applications, including superlenses, invisibility cloaks, and wideband artificial magnetic conductors. However, it has been proven that passive materials with these properties are necessarily narrow-banded and lossy. Active inclusions can be used to overcome these limitations, albeit at the expense of possible instability and noise issues, and added complexity. We examine a promising possible active implementation: the non-Foster circuit (NFC). Non-Foster circuits are used to create negative impedance loads, and have experienced widespread use for amplifying analog signals in trans-Atlantic telephony cables. There is also a long body of research into their applications for increasing antenna bandwidth. In this presentation, we demonstrate their application to multiple unit-cell metamaterials, and show their potential for possible performance improvements, including increased bandwidth and gain. We also show that maintaining stability is the primary difficulty in designing these complex systems, and has the effect of limiting the range of material properties that may be attained. Additionally, we examine noise and its effect as a mitigating factor in the performance of the metamaterial. Methods are demonstrated for designing robust, stable active metamaterials, and for analysing the noise figure of these structures. We conclude that, although noise and stability limits the bandwidths achievable with these active metamaterials, they still exhibit performances in excess of their passive counterparts, albeit at the expense of DC biasing-voltage requirements.

**Key words:** metamaterials, transformation optics, microwaves, dielectrics, negative impedance convertors.

### Introduction

Metamaterials are media for which the effective material properties are defined not by the constituent material properties, but rather by their substructured geometries. So although the materials within the meta-structure will have an effect on the overall properties, these effects are largely secondary to the unit-cell topology of the metamaterial. Examples of such metamaterials are wire media (1), which demonstrate negative-

permittivity (ENG) properties, and split-ring resonators (SRR) (2) which can exhibit negative-permeability (MNG). There has been particular excitement because these artificial materials can be used to achieve material properties that are not easily found in nature. The wire medium, for example, can be used to create a very high dielectric constant, or alternatively a near-zero or even negative dielectric constant. The SRR on the other hand, may similarly modulate the magnetic

permeability, without the need for magnetic materials such as ferrites.

There has been a large range of potential applications that have been suggested. Pendry suggested applying the negative refractive index – inherent when both permeability and permittivity are negative – to implement a ‘perfect’ lens that may beat the diffraction limit (3). Another intriguing application has been the invisibility cloak (4,5,6). An invisibility cloak serves to refract the incident electromagnetic waves such that they ‘bend’ around an object, thus rendering it invisible. We note that this technology differs markedly from ‘stealth’, in which the materials absorb and scatter the energy away from an observer.

The mathematics that makes this technology possible is called transformation optics (TO) theory. TO is a method of applying coordinate transformations to electromagnetics problems to simplify, or enable their solution (4,5). In effect, the problem is being warped such that it fits another, more easily solved geometry. Interestingly, in 2D it is simply a matter of selecting the most appropriate coordinate transformation. So a circle may be transformed to a half-space, but the materials will remain the same. However in 3D the problem becomes more complex as material tensors become involved. In this case, as the transformation is applied, then the material properties change. Thus, in order to solve the problem an entirely new range of materials (quite probably anisotropic and magnetic) are required.

Clearly, while the solution to the physics problem is now attainable (we now have invisibility cloaks!), the materials problem has become far more challenging. The required materials may have both electric and magnetic responses ( $\epsilon, \mu \neq 1$ ), and they will be both anisotropic and dispersive (in

frequency and space). It has been proven that passive materials cannot satisfy all conditions for a lot of TO devices (including free space invisibility cloaks) over a wide band of frequencies, and that they are largely very lossy (7,8). There are in effect three methods for tackling the problem of trying to develop these advanced TO-based devices:

1. Develop new materials.
2. Apply approximations to simplify the problem.
3. Incorporate active and nonlinear components.

The first approach has involved the development of a greater range of materials with lower losses and more practical applicability. While piezoelectric or ferroelectric materials, for example, may be used to invoke various tunable properties within the structure, the steady-state performance of passive structures is still narrowband.

Certain approximations have been applied with great success in the application of transformation optics, in order to reduce the level of anisotropy and dispersion, and therefore narrow the range of required material properties. As an example, the ground-plane quasicloak was developed. For this cloak, anisotropy can in practice be reduced to such a level that it is almost negligible, and can thus be ignored in implementation. Perhaps more importantly, if the cloak is to be targeted at a single polarization then the material span can be purely dielectric ( $\mu = 1$ ) and with permittivity greater than air ( $\epsilon \geq 1$ ). Finally, spatial dispersion can be overcome by discretizing the space into blocks, as in Figure 1. The great benefit of this technique is that it may be implemented using all real materials. The authors have demonstrated a realization of this approach in (6), based on loading a

polyurethane foam with a  $\text{BaTiO}_3$  powder at varying densities to create the matrix of required dielectrics.

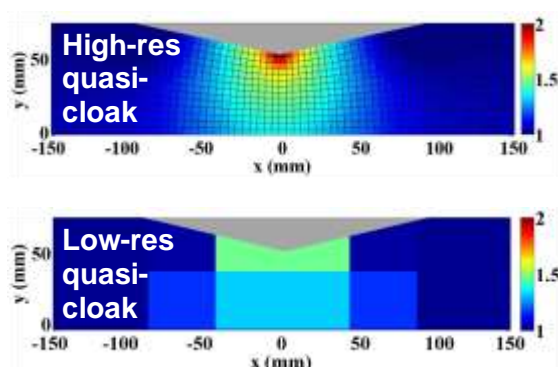


Figure 1 - Dielectric map of high- and low-resolution carpet cloaks.

However, while there are certainly practical applications of this second approach it is limited in that it is only applicable to certain geometries. It can only be applied to the ground-plane quasicloak for example, and not to a free-space version. The reason for this is because the propagating wave must have a faster-than-light phase velocity in the free-space version, requiring frequency-dispersive materials. The third approach is targeted at these issues: using active components to overcome the limitations of passive materials.

In this paper, we will discuss a method we have applied with the aim of increasing the bandwidth and reducing the losses of an effective medium. This approach involves loading the metamaterial with a non-Foster circuit. We will describe the nature of this circuit, as well as the primary challenges and limitations we face in its implementation.

### Non-Foster Circuits and Applications

In the 1950's, as solid-state negative resistance circuits were developed to replace vacuum tubes for the purpose of analog signal amplification (9). These circuits became particularly prevalent for boosting signals along the trans-Atlantic telephony cables, and

were in widespread use until the eventual conversion to digital technologies in the 1970's. It was similarly recognised that Linvill's circuits could be used to create negative complex impedances (resistance and reactance), to enable the creation of negative capacitances and inductances. The circuit would operate by inverting the current (voltage) while the voltage (current) remains unchanged, and was named the negative impedance converter (NIC).

An exciting application of the NIC was the promise of broadband impedance matching of electrically-small antennas (ESA) (10). The idea behind this technique is shown in Figure 2. An electrically-small antenna (such as a short monopole) is represented as a positive capacitance  $C$ . An important part of impedance matching is the cancellation of the reactance. As the reactance of a capacitance is negative, and given as  $X_{ant} = -j/\omega C$  we aim to cancel this using a positive reactance. A passive inductor is the common solution, with a reactance of  $X_L = j\omega L$ . However it is clear to see that the resultant equivalent circuit will cancel reactance only at the resonant frequency of  $\omega_0 = 1/\sqrt{LC}$ . An active negative capacitor, given by  $X_{-C} = +j/\omega C$  will, on the other hand cancel the reactance of the antenna over an arbitrarily large frequency range. This idea would suggest the possibility of wideband impedance matching of ESAs.

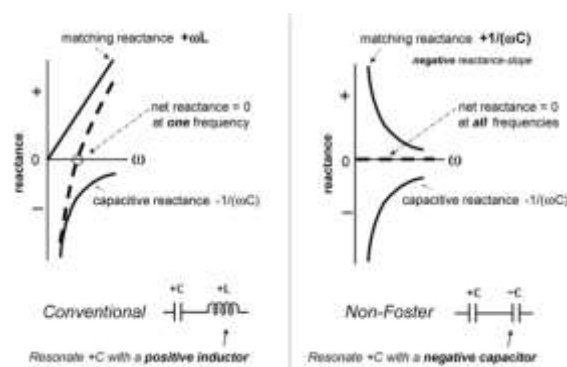
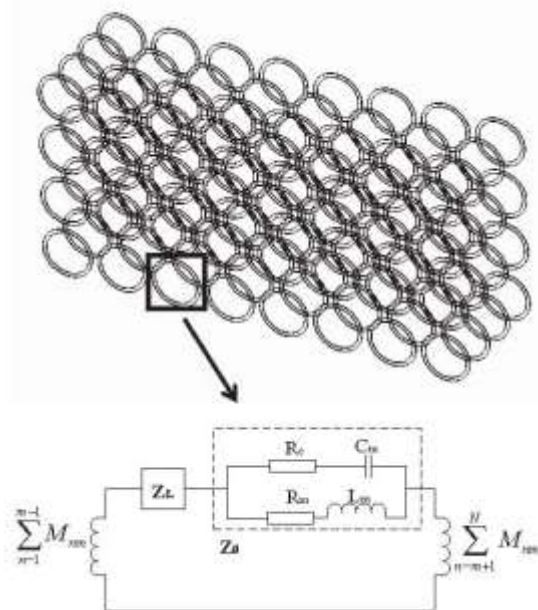


Figure 2 - Impedance matching of an electrically-small antennas using passive  $+L$  (left) and active  $-C$  (right) loads.

Naturally, there are limitations to this technology that will restrict the performance of this application. Firstly, irrespective of the nature of the impedance matching method, the performance of the antenna will be limited by the radiation bandwidth. There are also fundamental physical limits that are inherent to active technologies: stability and noise. The overall structure must be designed such that stability is maintained. This implies that the negative capacitance cannot overcompensate for the antenna's positive capacitance, and indeed a practical system must include a safety buffer to avoid possible instability due to parasitics. Furthermore, the introduction of further electronic circuitry may lead to higher levels of Nyquist-Johnson noise.



**Figure 3 - A metamaterial made of NIC-loaded loops (above) along with its equivalent circuit (below).**

In Figure 3 we show the implementation of a metamaterial with enhanced magnetic response over a broad band of frequencies. An NIC is used to load each unit cell of the metamaterial in order to compensate for inductive effects of the loops, as well as the mutual coupling between different elements. The overall effect is that the dispersion of the

magnetic permeability profile is significantly reduced, and  $\mu$ -near-zero (MNZ) or  $\mu$ -negative (MNG) properties are possible over a large range of frequencies (11,12).

The stability criteria for these structures were investigated in detail, and it was demonstrated that stability may in fact be maintained, however at the expense of a certain level of dispersion. Nevertheless, the level of dispersion is far lower than that of a passive resonant metamaterial. This has the potential to increase the bandwidth of structures such as invisibility cloaks and high-impedance surfaces.

The Johnson-Nyquist noise within the structure can also be investigated. In Figure 4 we compare the noise figure in loops for different scenarios. First we compare the noise figure between passive loops, and active loops with varying negative inductance. Clearly the passive loop will have a low noise figure, however it is clear that as the loop approaches resonance, the noise figure of the active loop increases significantly. We can see a similar trend as the number of loops is increased. This will have repercussions as bulk metamaterial and multiple unit-cell applications are developed, as the high noise figure will lead to a reduction in performance. Nevertheless, the improved material performance and potential for gain and low dispersion will largely override these disadvantages, as long as care is taken in the engineering design.

## Conclusions

We have discussed approaches for the improvement of metamaterial properties. In particular, we have discussed the application of active electronics to the reduction of dispersion in metamaterials. Despite potential instability and high noise figures, the performance shows significant improvement,

with potential applications to wideband magnetic metamaterials, TO devices such as invisibility cloaks and flat lenses, and to the development of improved artificial magnetic conductors, high impedance surfaces, and other such structures.

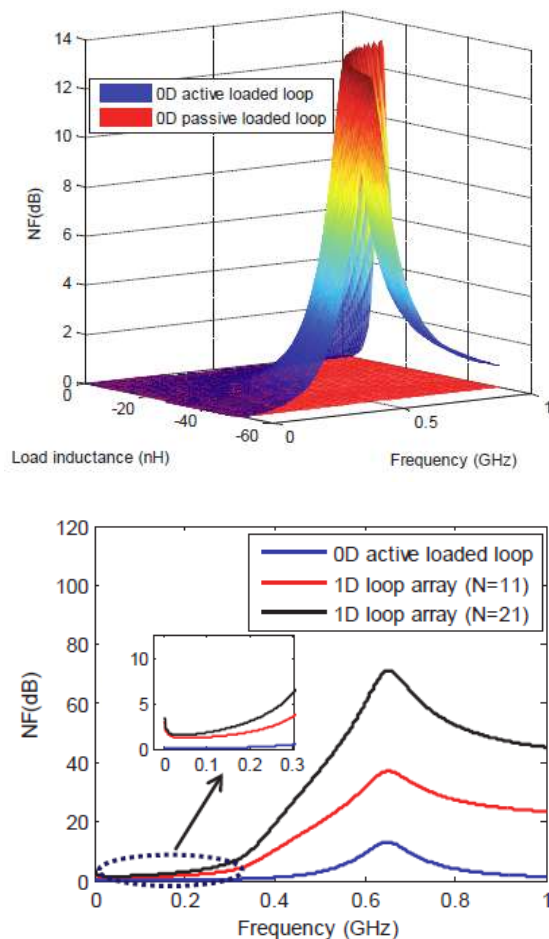


Figure 4 - Noise figure within a single passive/active loop (above) and along a 1D line of coupled loops (below).

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