A Curvature-Based Interpretation of the Steinberg Criterion for Fatigue Life of Electronic Components

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Abstract

The "Steinberg Criterion" is a well-known method for determining the fatigue life of Printed Circuit Board (PCB) components based on the deflection of the PCB. It has been adopted as a de facto industry standard for the fatigue analysis of electronic components, and has been successfully used on many programs. However it has some limitations. Steinberg derived this equation to describe the behavior of rectangular PCBs simply supported on all sides. In this configuration the deformed shape of the first mode of a PCB under vibratory loads is assumed by Steinberg to be described by two perpendicular half sine waves. Unfortunately many PCBs have distorted mode shapes as a result of clamped or asymmetric edge constraints, stiffeners, or irregular PCB outline. Finite Element Models (FEMs) can be used to predict mode shapes for these PCBs, but there has been no clear way to use Steinberg's equation to determine the fatigue margin for components on such boards. The traditional method (when the discrepancy is addressed) is to use a value for PCB length in the equation based on an approximation of the length of an equivalent half sine wave superimposed on the predicted mode shape. This approach, while better than ignoring the problem, can lead to inconsistency in results or the overlooking of localized effects, and in the case of extremely odd mode shapes can be nearly impossible.

This paper presents a method of using FEM data for curvature as well as deflection at a single location to eliminate the shape and location variables from the Steinberg criterion, allowing it to be applied confidently to PCBs and Printed Wiring Assemblies (PWAs) with any shape and boundary conditions. Test cases are described that show equivalence between this method and the existing Steinberg criterion. Lastly the methodology used to extract phase-consistent curvature and deflection results from FEM analysis is briefly discussed.

Key words

Analysis, Curvature, Fatigue, PCB, Steinberg

I. Introduction

Most engineers involved in electronics packaging and circuit board design are by now familiar with equation 8.34 from Dave S. Steinberg's book, Preventing Thermal Cycling and Vibration Failures in Electronic Equipment [1]:

$$\delta_{max} \le \frac{.00022 * B}{C * h * r * \sqrt{L}} \qquad (1)$$

Where:

δ_{max} = Maximum normal displacement of the PCB, in inches;

B = Length of Printed Circuit Board (PCB) edge parallel to the long edge of the component, in inches;

C = constant for different types of electronic components;

h = height or thickness of the PCB, in inches;

r = Relative position factor for location of components on the PCB;

L = length of the rectangular component, in inches.

Known colloquially as "The Steinberg Criterion," this has been adopted as a de facto industry standard for the fatigue analysis of electronic components, and has been successfully used on many programs [2], [3]. However it has some limitations [4]. Steinberg derived this equation to describe the behavior of rectangular PCBs simply supported on all sides (see fig.1).

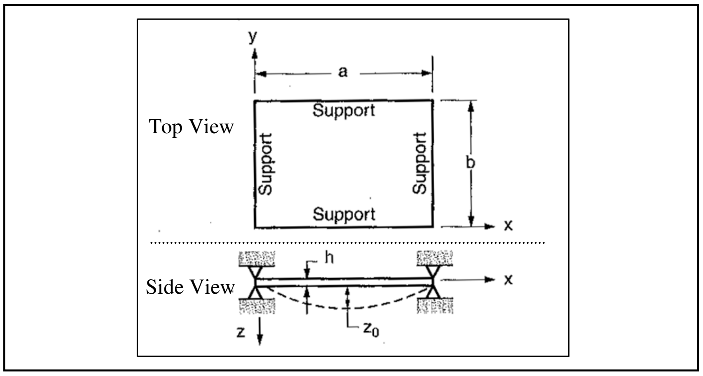

Fig. 1. (Fig. 4.1 from [1]) Assumed PCB Configuration

In this configuration the deformed shape of the first mode of a PCB under vibratory loads is assumed by Steinberg to be described by two perpendicular half sine waves.

Unfortunately many PCBs have distorted mode shapes as a result of clamped or asymmetric edge constraints, stiffeners, or irregular PCB shape. Finite Element Models (FEMs) can be used to predict mode shapes for these PCBs, but there has been no clear way to use Steinberg's equation to determine the fatigue margin for components on such boards. The traditional method (when the discrepancy is addressed) is to use a value for B in the equation based on an approximation of the length of an equivalent half sine wave superimposed on the predicted mode shape. This approach, while better than ignoring the problem, can lead to inconsistency in results or the overlooking of localized effects, and in the case of extremely odd mode shapes can be nearly impossible.

Presented here is a method of using FEM data at a single location to eliminate the shape and location variables (B and r) from the Steinberg criterion, so that it can be applied confidently to PCBs and Printed Wiring Assemblies (PWAs) with any shape and boundary conditions.

II. Elimination of r; Using Data at Multiple Points

As originally written the Steinberg Criterion applies to the maximum displacement at the center of the PCB. To make the results applicable to components that reside elsewhere on the board Steinberg introduces the position factor, r, to account for the reduced deflection seen at locations away from the center. When using an FEM, which easily gives deflections at many locations on the PCB, it is common engineering practice to always set r=1 and compare the allowable thus calculated to the deflection at the component location. In this section it will be proven that the two approaches are exactly equivalent.

From equation 4.2 and section 8.5 in [1] it can be shown that:

$$r = \sin\left(\frac{\pi X}{a}\right)\sin\left(\frac{\pi Y}{b}\right) \tag{2}$$

Where X, Y, a and b are as shown in fig. 1.

For a deflected board whose shape is described by two perpendicular half sine waves, δ at (X,Y) can be found by the expression:

$$\delta = \delta_{max} * \sin\left(\frac{\pi X}{a}\right)\sin\left(\frac{\pi Y}{b}\right) \tag{3}$$

Substituting (2) into (3) gives:

$$\delta = \delta_{max} * r \tag{4}$$

Now taking (1) and multiplying both sides by r produces:

$$\delta_{max} * r \le \frac{.00022 * B}{C * h * \sqrt{L}} \tag{5}$$

Substituting (4) into (5) results in:

$$\delta \le \frac{.00022 * B}{C * h * \sqrt{L}} \tag{6}$$

Thus setting r = 1 and comparing the resulting allowable to the deflection at the component location is shown to be exactly equivalent to (1).

III. Derivation of B from Deflection and Curvature Data at a Point

As stated previously the Steinberg criterion was derived assuming a rectangular PCB simply supported on all sides. Thus the criterion is applicable with perfect correctness to this configuration. For simplicity, consider a one-dimensional system. The equation for the deflection at any point is:

$$\delta = \delta_{max} * \sin\left(\frac{\pi X}{B}\right) \tag{7}$$

Given (7), the curvature, κ, can be found by taking the second derivative of δ with respect to X:

$$\kappa = \delta_{max} * \left(\frac{\pi^2}{B^2}\right)\sin\left(\frac{\pi X}{B}\right) \tag{8}$$

(Note that signs are ignored.) Dividing (7) by (8) gives :

$$\frac{\delta}{\kappa} = \left(\frac{B^2}{\pi^2}\right) \tag{9}$$

Solving for B:

$$B = \pi\sqrt{\frac{\delta}{\kappa}} \tag{10}$$

By setting r=1 and using local deflection data, and using (10) to substitute for B, the Steinberg Criterion (1) can be written as:

$$\delta \le \frac{.00022 * \left[\pi \sqrt{\frac{\delta}{\kappa}}\right]}{C * h * \sqrt{L}} \quad (11)$$

Rearranging the expression renders:

$$\sqrt{\delta * \kappa} \le \frac{.00069}{C * h * \sqrt{L}} \quad (12)$$

This expression meets the goal of having a criterion that eliminates shape and location data. The "allowable" on the right side of the inequality depends solely on L and C, characteristics of the component being analyzed and h, the thickness of the PCB. The left side depends only on data obtainable at every point from FEM results. Subsequent sections will verify this expression against (1) by using both to analyze cases for which the answers are known.

IV. FEM (Finite Element Model) Verification

All verification cases were analyzed for a typical PWA component, a CWR09 surface mount capacitor, case size A. For this component:

c=2.25; L(x)=0.00254 m (0.10 in.); L(y)=0.00127 m (0.05 in.)

All verification cases assume a simple Printed Circuit Board (PCB) of weight 8.45 N (1.9 lb.), height 0.1524 m (6.0 in.), width 0.254 m (10.0 in.) and thickness, h, of 0.00236 m (0.093 in.). Three of the many cases run are presented here.

A. PCB Simply Supported on Four Sides

The FEM was run using NASTRAN SOL 103 (Normal Modes). The results were then scaled for a maximum deflection (δ_{max}) of 0.00061 m (0.024 in.), equivalent to that of a PCB subjected to roughly 50 Gs. The deflected mode shape can be seen in fig. 2.

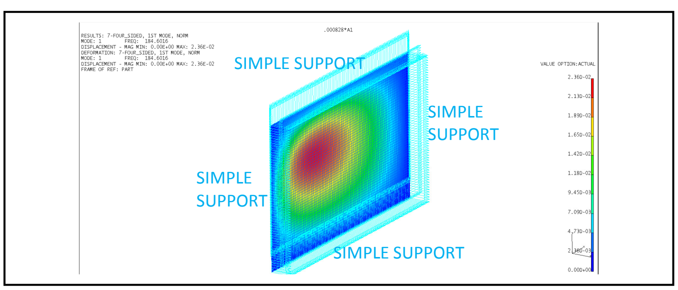

Fig. 2. First Mode of PCB Simply Supported on All Sides

An analysis was then done for a typical PWA component, a CWR09 surface mount capacitor, case size A. This part was assumed to be mounted at four locations on the PWA, as seen in fig 3. In all cases the long edge of the part was aligned with the long edge of the PWA.

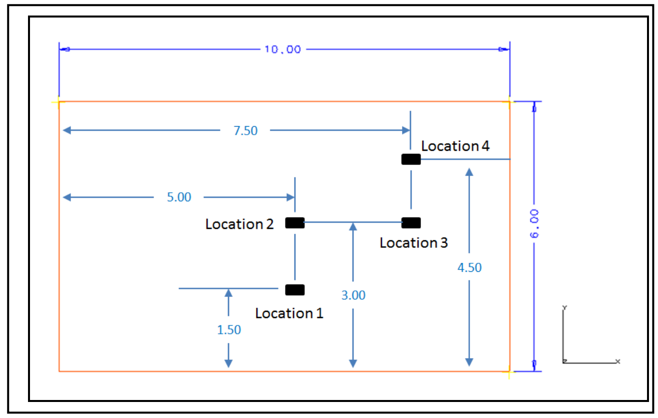

Fig. 3. Component Locations (Dimensions in inches)

Equations (1) and (12) were used to compute Margins of Safety (MS) for the components according to the following formula:

$$MS = \frac{Allowable}{Actual} - 1 \quad (13)$$

Note that Steinberg assumes all dimensions are in inches.

Variables and results for each component location are listed in Table I.

TABLE I. VERIFICATION RESULTS, CASE A

Location	Curvature Direction	L, m (in.)	B, m (in.)	r	δ, m (in.)	κ, m^{-1} (in.$^{-1}$)	M.S. using (1)	M.S. using (12)
1	RX	0.00127 (0.05)	0.152 (6.00)	0.707	0.000424 (0.0167)	0.180 (0.00456)	0.69	0.69
2	RX	0.00127 (0.05)	0.152 (6.00)	1.00	0.000599 (0.0236)	0.254 (0.00646)	0.19	0.19
2	RY	0.00254 (0.10)	0.254 (10.00)	1.00	0.000599 (0.0236)	0.091 (0.00232)	0.41	0.41
3	RY	0.00254 (0.10)	0.254 (10.00)	0.707	0.000424 (0.0167)	0.065 (0.00165)	0.99	0.99
4	RX	0.00127 (0.05)	0.152 (6.00)	0.50	0.000300 (0.0118)	0.128 (0.00324)	1.39	1.38
4	RY	0.00254 (0.10)	0.254 (10.00)	0.50	0.000300 (0.0118)	0.046 (0.00117)	1.81	1.81

Equation (10) was used to estimate B in the X and Y directions using FEM output data. Results are shown in figures 4 and 5 for X and Y.

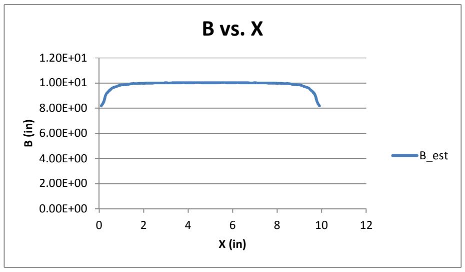

Fig. 4. B in X Direction Computed from FEM Output (in inches)

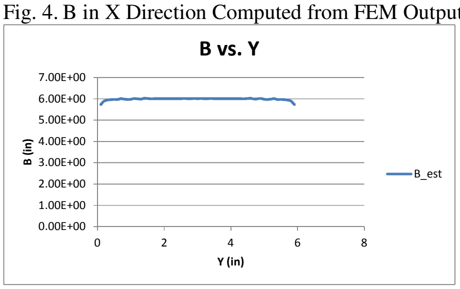

Fig. 5. B in Y Direction Computed from FEM Output (in inches)

Margins of Safety show near perfect agreement between (1) and (12). The small differences are likely due to rounding errors. The B estimates for X and Y agree well with the expected values of 0.254 m (10.0 in.) and 0.152 m (6.0 in.), respectively, except near the edges where the estimated B is a bit low. This may be due to FEM boundary effects.

B. PCB Simply Supported on Two sides, Higher Mode

The FEM, simply supported along its long (X) edges, was run using NASTRAN SOL 103 (Normal Modes). The deflected shape considered was an S shape, as might be seen in a PCB restrained along its centerline or excited by a higher frequency resonance. Deflections were scaled for a maximum deflection of 0.000673 m (0.0265 in.) The deflected mode shape can be seen in fig. 6.

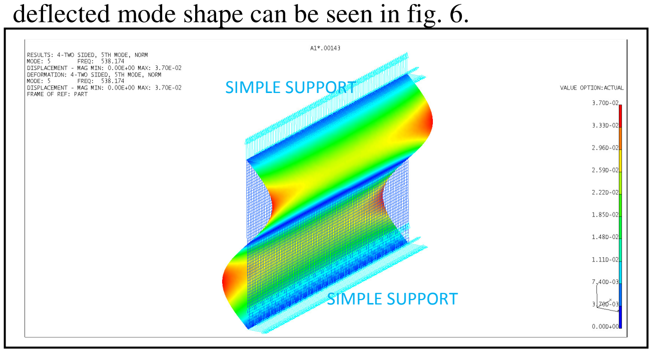

Fig. 6. Fifth Mode of PCB

Because the mode shape clearly divides the PCB into two equal portions, it is reasonable to assume that a B value of 0.0762 m (3.0 in.) would be used for analysis.

An analysis was then done for the same typical PWA component. This part was assumed to be mounted at two locations on the PWA, as seen in fig 7.

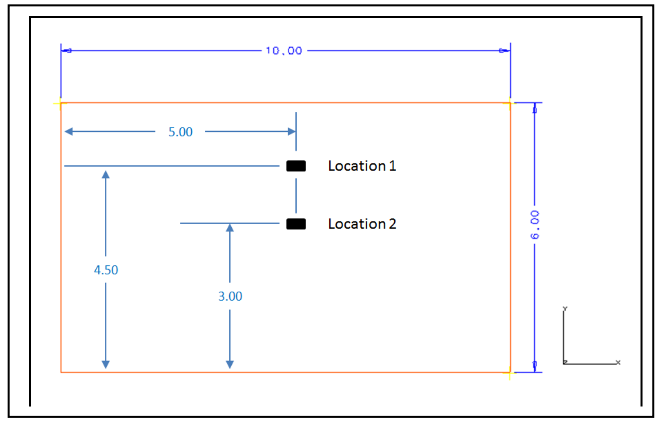

Fig. 7. Component Locations (Dimensions in inches)

Equations (1) and (12) were used to compute Margins of Safety (MS) for the components according to (13). Variables and results per location are listed in Table II.

TABLE II. VERIFICATION RESULTS, CASE B

Location	Curvature Direction	L, m (in.)	B, m (in.)	r	δ, m (in.)	κ, m^{-1} (in.$^{-1}$)	M.S. using (1)	M.S. using (12)
1	RX	0.00127 (0.05)	0.076 (3.00)	1.000	0.000673 (0.0265)	1.140 (0.0289)	-0.47	-0.47
2	RX	0.00127 (0.05)	0.076 (3.00)	0.00	0.00 (0.00)	∞	∞	∞

Note that, if the mode shape had not been observed and the PCB analyzed using the full height for B, (1) would give margins of +0.51 and +0.06 at Locations 1 and 2 respectively. This would not occur if (12) were used, since analyst judgment about r and B would not be needed to produce correct answers.

Equation (10) was used to estimate B using FEM output data. Results are shown in fig. 8.

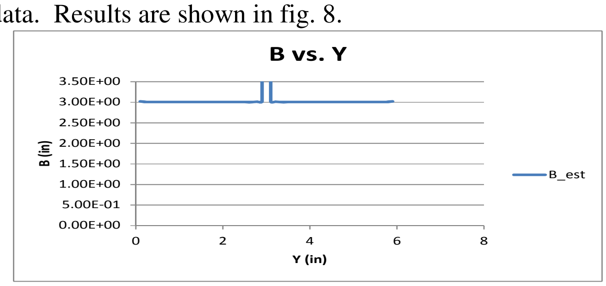

Fig. 8. B Computed from FEM Output (in inches)

In either "half" of the PCB the effective B is found to be 0.0762 m (3.0 in.), except for the center where B effectively becomes infinite, denoting a small, virtually flat region.

C. PCB Clamped on Four Sides

The FEM was run using NASTRAN SOL 103 (Normal Modes). The results were then scaled for a maximum deflection (δ_{max}) of 0.00016 m (0.0062 in.), equivalent to that of a PCB subjected to roughly 50 Gs. The deflected mode shape can be seen in fig. 9.

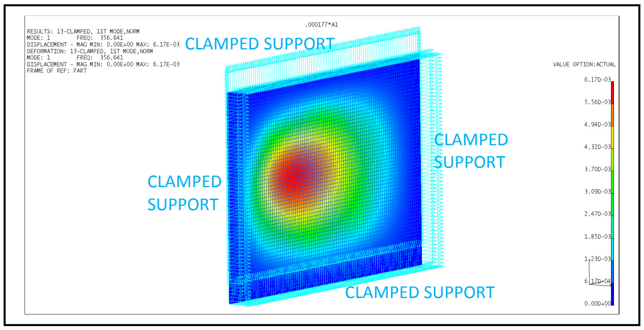

Fig. 9. First Mode of PCB Clamped on Four Sides

This common and seemingly simple configuration nonetheless makes application of the Steinberg criterion difficult. A cross section of the mode shape in the YZ plane (shown in fig. 10) is clearly not a half sine wave. An experienced and careful analyst might assume an effective B somewhat smaller than the full 0.152 m (6.0 in.). Most often the edge fixity would probably be ignored and the PCB analyzed as if it conformed to Steinberg's assumptions. That is what will be done for this example.

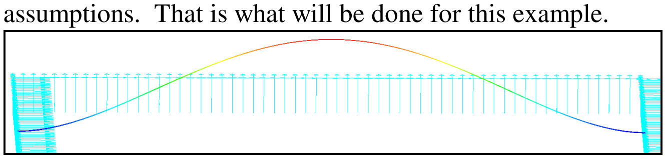

Fig. 10. Cross Section of First Mode of PWA

An analysis was then done for the same typical PWA component. This part was assumed to be mounted at three locations on the PWA, as seen in fig 11.

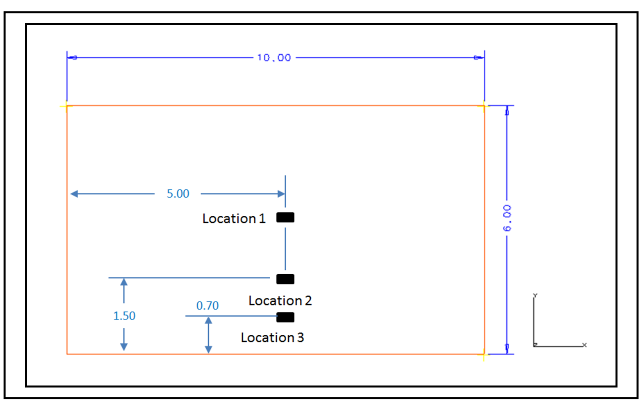

Fig. 11. Component Locations (Dimensions in inches)

Variables and results per location are listed in Table III.

TABLE III. VERIFICATION RESULTS, CASE C

Location	Curvature Direction	L, m (in.)	B, m (in.)	r	δ, m (in.)	κ, m^{-1} ($in.^{-1}$)	M.S. using (1)	M.S. using (12)
1	RX	0.00127 (0.05)	0.152 (6.00)	1.000	0.000157 (0.0062)	0.112 (0.0028)	3.57	2.52
2	RX	0.00127 (0.05)	0.152 (6.00)	0.707	0.000087 (0.0034)	0.022 (0.00056)	5.46	9.67
3	RX	0.00127 (0.05)	0.152 (6.00)	0.36	0.000026 (0.0010)	0.085 (0.0022)	26.90	9.00

These results cannot truly be considered verification, since the "correct" answer according to Steinberg is not known. If the revised criterion presented here is to be believed this does indicate that the margins for parts at the center or near the edges of a PWA with this common configuration are likely to have been assigned margins that were too high. Given the shape of the deformed PWA, with its greater degree of curvature near the center than would be the case in a simply supported PWA, this conclusion seems intuitively correct.

Equation (10) was used to estimate B using FEM output data. This shows that in the central region of the PWA the effective B is a bit smaller than the full height of the PWA, which makes sense. It also shows that there are inflection points in the mode shape at roughly 0.038 m (1.5 in.) from either boundary where the effective B becomes large. Also the regions near the boundaries show small values for effective B, indicating that while deflections in these regions are low the curvatures are high. This may mean that putting sensitive components near an edge may not automatically be safe. Results are shown in fig. 12.

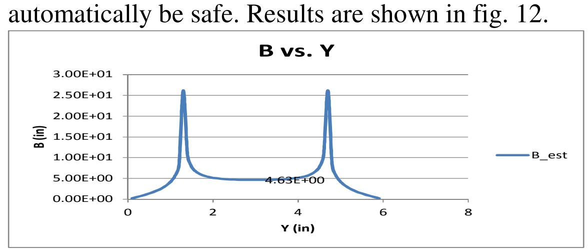

Fig. 12. B Computed from FEM Output (in inches)

V. Computing Methods

Using this method for random vibration analysis is more difficult because any RMS or Spectral Density of the combined criterion of $\sqrt{(\kappa*\delta)}$, a.k.a. RCxD, should be calculated using phase consistent combinations of components. Ball Aerospace engineers have developed a MATLAB based computer code that reads OUTPUT2 files and obtains the RCxD Spectral Density and RMS quotient due to random vibration by adapting the method used for obtaining Von Mises Spectral Density described in [5].

VI. Conclusion

It has been shown that a revised statement of the Steinberg criterion can be made which eliminates reference to the location of the component and the shape of the overall PCB. Doing so can eliminate analysis ambiguities associated with PCBs which do not conform to the shape and boundary assumptions used by Steinberg. Use of this criterion requires knowledge of deflection and curvature at the component location. This information can be extracted from FEM results.

This method is still not flawless. For example a situation can be imagined where a component sits at an inflection point with zero displacement and high curvature. Intuitively this component would be highly stressed, but this criterion would predict an infinite margin of safety (since $\delta=0$). The original Steinberg criterion would also make the same mistake. To eliminate this the perfect criterion would be based solely on curvature. However since the existing database of Component Constants (C) was derived using δ as the correlated variable a entirely new database of constants would have to be determined empirically using κ as the correlated variable. The "root deflection times curvature" or "RCxD" criterion described here is mathematically equivalent to the original Steinberg criterion (and thus can use the same C constants) and does take curvature into account, even if imperfectly.

Acknowledgment

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