

Design of Multiwall Carbon Nanotube Based Embedded Inductors

Bruce C. Kim

University of Alabama
Electrical and Computer Engineering
Tuscaloosa, AL 35487
U.S.A.

ABSTRACT

Carbon nanotubes enable us to replace conventional metal to achieve high-value inductors with high quality factors. The values of inductance and quality factors are critical to high-quality RF product design. Current spiral inductors are widely used in many RF circuits such as cell phones and wireless routers. It is desirable to achieve low power, low phase noise, and wide bandwidth wireless systems with high Q inductors. This paper describes the design and simulation of Multiwall Carbon Nanotube (MWCNT) based stacked inductors.

Index Terms—Inductors, Multi-Wall Carbon Nanotube, quality factor, substrate, stacked inductors.

1. INTRODUCTION

Today's high volume RF products require high inductance and high Q factor RF inductors. There is a great deal of open literature on the design of radio frequency (RF) inductors on multilayer structures [1-4]. The spiral inductors are designed to be stacked on different layers to increase the mutual coupling between the inductors that are stacked. This type of design usually increases the overall inductance. Conventional RF inductors use spiral-shaped metal on a single layer of the substrate, and they usually have a large footprint with limited inductance and Q (quality) values. Using such large-footprint inductors on a silicon substrate, as an example, inhibits adding more transistors on the given silicon area. Carbon nanotubes enable us to replace conventional metal to achieve high-value inductors and quality factors. The values of inductance and quality factors are critical to high-quality RF product design. Current spiral inductors are widely used in many RF circuits such as in transceivers for cell phones and wireless routers [5-8].

2. CARBON NANOTUBES

Nanotechnology-based package products have recently emerged in the semiconductor industry. Among the nanotechnology-based products on the market, carbon nanotubes are one of the most promising materials for realizing nanoscale inductors. Carbon nanotubes consist of a honeycomb lattice of a graphene sheet rolled into a cylinder. Multiwall CNT (MWCNT) and Single-Wall (SWCNT) are two types of CNT available on the market. They are several micrometers in length and a few nanometers in diameter, resulting in a large aspect ratio that allows approximation of

the carbon nanotubes as an **1D** system. It is interesting to note that carbon nanotubes can be classified as a metallic or semiconductor; their characteristics can be changed by adjusting the chirality and diameter of CNT. Pressing or stretching CNTs can change their electrical properties by changing the quantum states of the electrons in the carbon bonds. It is envisaged that one-third of rolled CNTs will be metallic and two-thirds will be semiconductor [9, 10]. Both single-wall and multiwall carbon nanotubes have a low resistivity of approximately $150 \mu\Omega\cdot\text{cm}$. MWCNTs can carry electrical current densities up to 10^{10} A/cm^2 and remain stable for an extended period of time at high temperatures. Although CNTs can carry very high current densities, CNTs on a metal contact may measure high contact resistance in the order of $k\Omega$. The resistivity of CNTs is approximately $10^{-7} \Omega\text{m}$. This makes an ideal material for RF inductors.

Schrödinger Equation

Quantum mechanical effects begin to dominate in nano-scaled carbon nanotubes as their size shrinks. To model carbon nanotubes, we consider kinetic and magnetic inductance. These inductance values are derived from solving both Schrodinger and Boltzmann transport equations. The time-independent Schrödinger equation (Eqn1) in one dimension and the Boltzmann transportation equation (Eqn2) are used to calculate the values of quantum capacitance, quantum (intrinsic) resistance, and kinetic inductance for the spiral inductor made from MWCNT. In steady state, an equal number of electrons is moving in positive and negative **k** direction. CNT is an **1D** system with limited density of states. With potential applied, the carriers have to move to higher energy states resulting in increase in kinetic energy. Extra stored energy results in kinetic inductance. CNTs have two type of capacitance: quantum capacitance and electrostatic capacitance. The quantum is due to the potential energy variation from state to state.

The Boltzmann transport equation is used to analyze transport phenomena within systems that involve density and temperature gradients. The equation is applied to the analysis of the broad-spectrum of currents within a system [12-15]. Solving both the Schrodinger and the Boltzmann transport equations will yield equations (4, 7, and 9) for kinetic inductances, quantum capacitance, and quantum resistance.

$$\frac{d^2\psi(x)}{dx^2} + \xi[E]\psi(x) = \xi\lambda\psi(x) \quad \text{Eqn(1)}$$

$$f(x, v, \phi) = f_0(x, v) - I \cos(\phi) \left[\frac{\partial f_0}{\partial x} + \frac{a}{v} \frac{\partial f_0}{\partial v} \right] \quad \text{Eqn(2)}$$

Assume that there is a time-dependent electric field only in the x-direction and confinement in the y and z directions. The kinetic inductance is derived in Equation (3).

$$L_k = \frac{h}{2e^2 v_F} \quad \text{Eqn(3)}$$

The quantum Resistance can be extracted from partial differential equation in Equation (4).

$$\frac{\partial I}{\partial t} + \frac{I}{\tau} = \frac{e}{L_k} \frac{\partial V}{\partial x} \quad \text{Eqn(4)}$$

Solving Equation (4) gives us the quantum resistance and capacitance:

$$R = L_k / \tau = \frac{h}{4e^2} \quad \text{Eqn(5)}$$

$$C_Q = \frac{2e^2}{h v_F} \quad \text{Eqn(6)}$$

where h is the Planck constant, e is the electron charge, v_F is the Fermi velocity, τ is the Relaxation time, L_k is the kinetic inductance, and C_Q is the quantum capacitance.

3. APPROACH

A conventional integrated inductor is made from aluminum metal with a specific turns ratio, thickness, diameter, and spacing. Our proposed CNT based inductor consists of a substrate and a multiwall CNT. We propose to use multiwall CNT's on a silicon substrate as shown in Figure 4. The figure shows a spiral inductor where each segment is made of carbon nanotube. The unique feature about this design is that the CNT-based inductors are designed on two-layer where an insulator separates the two.

Circuit Models for MWCNT Stacked Inductors

The inductor is an indispensable part of many RF circuit designs. The stacked two-layer carbon nanotube inductor shown in Figure 1 is modeled as an electrical equivalent circuit as shown in Figures 2. In the first layer, each segment of the carbon nanotube forms a parallel capacitor and inductor with the silicon substrate, labeled as C_{Sub} and R_{Sub} . The capacitor C_{ox} represents the capacitance formed with silicon dioxide layer shown in Figure 1. In MWCNT each wall has resistor R_Q (Quantum or Contact resistor). Each MWCNT contains kinetic inductance, self inductance and mutual inductance which are labeled as L_k , L_s , and L_{Mx} , respectively. MWCNT inductor forms three types of capacitances. The electrostatic capacitance (C_E) is formed by each segment of the carbon nanotube to the reference plane,

the quantum capacitance (C_Q) is due to quantum effect, and the coupling capacitance (C_{cx}) is formed between two walls of the carbon nanotubes.

A number of lumped element models have been previously published [1,2,4,8,12,16]. Some researchers have modeled inductors and analyzed them on a turn-by-turn basis. Others have analyzed inductors on a segment-by-segment basis of the metal. In this paper, our analysis of the CNT inductor will be performed segment by segment on a multiwall carbon nanotube.

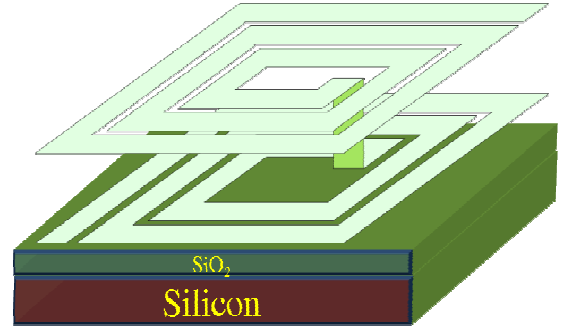


Fig. 1. Stacked Two-Layer Multiwall Carbon Nanotube Inductor on Silicon Substrate.

When inductors are stacked as shown in Figure 1, the total inductance is increased by n^2 where n is the number of layers. However, stacking of inductors suffers from lower self-resonant frequency due to interlayer capacitances. The CNT spiral stacked inductor discussed in this paper has a 2.5 turns ratio. We used a substrate thickness of 200 microns for silicon and 5 mils for ceramic substrates. The two parallel branches represent parasitics on port-1 and port-2. Figure 2 can be modified to accommodate a ceramic substrate by removing the oxide capacitance, C_{ox} . A series inductance and resistance represent the segment of each CNT stacked inductor. As the different segments are close to each other, there is a mutual inductance between the segments. The mutual inductance and the coupling coefficients between the different self-inductances are calculated by finding the magnetic field inside the spiral CNT segment. These mutual inductances are represented in the model as L_{Mx} .

The circuit model in Figure 2 is unique due to quantum effects such as quantum capacitances and kinetic inductances. Other models that have been proposed in the literature for CNT provided the inductor model as a transmission line [16]. Our models take into consideration kinetic and quantum effects that arise from the decrease in size and the high aspect ratio properties of MWCNT.

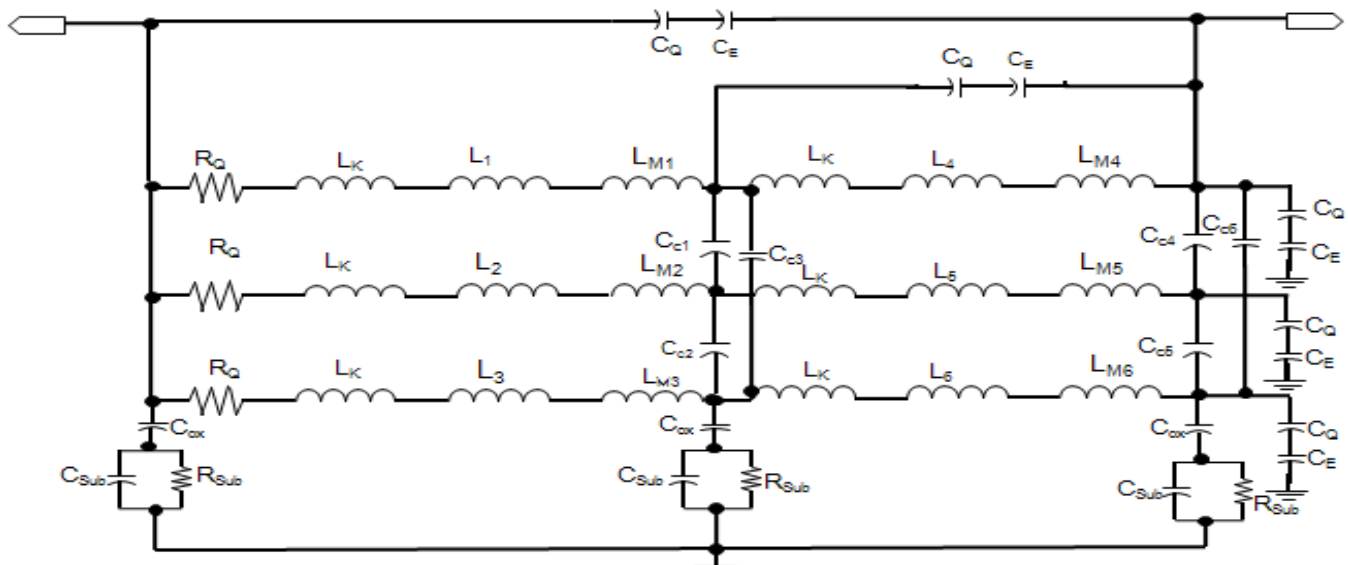


Fig. 5. Equivalent Circuit Model of Two-Port MWCNT Stacked Inductor on a Silicon Substrate.

A simple definition of inductance (L) is the property of a conductor that describes the proportionality between current change and induced voltage. An inductor is any conductor across which there is a voltage drop when a time-varying current is present. Kinetic inductances in MWCNT are more than four orders of magnitude larger than in metal spiral inductors [12]. Our MWCNT stacked inductor is divided into eleven segments each layer. Each MWCNT segment is 125nm long and 25nm in diameter. The magnetic inductance of each segment can be calculated by considering the MWCNT as a cylinder 25nm in diameter and 125 nm in length. These dimensions have been used in Equation (7) to calculate inductances.

$$L = \frac{\mu_0 l}{2\pi} \ln \left(\frac{d_2}{d_1} \right) \quad \text{Eqn(7)}$$

Electrostatic Capacitance

Capacitance (C) is determined by the length and cross-sectional dimension of MWCNT. The electrostatic capacitance of the MWCNT can be computed using the cylindrical formula for capacitance in Equation (8).

$$C_E = \frac{2\pi\epsilon}{\ln\left(\frac{y}{d}\right)} \quad \text{Eqn(8)}$$

While MWCNTs have desirable material properties, individual CNT suffers from a large intrinsic resistance that is not dependent on the length of the CNTs. To alleviate the intrinsic resistance problem, bundles of CNTs have been used.

4. FABRICATION

As an initial demonstration, we have begun to fabricate a single-wall carbon nanotube (SWCNT) that is approximately 100nm long using the E-beam lithography technique. Figure 3 shows the ultra high-resolution E-beam pattern used to grow carbon nanotubes. These patterns are approximately 18nm in size. The carbon nanotubes are grown in our special CVD (Chemical Vapor Deposition) chamber. Using the same techniques for SWCNT, we have demonstrated the growth of a single MWCNT with 125 nm length and 25nm diameter.

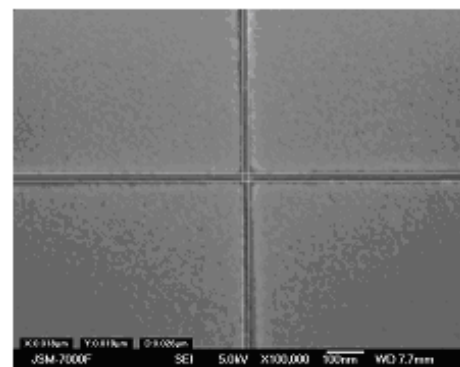


Fig. 3. Sample Electron Beam Lithography Patterns Showing Ultra-High Resolution of Nickel Islands.

Figure 4 illustrates approximately 100nm long SWCNT interconnect between two 30nm electron beam nickel islands. These nickel islands were formed by ultra-high resolution electron beam lithography as shown in Figure 3. We have grown the carbon nanotubes in our special CVD chamber.

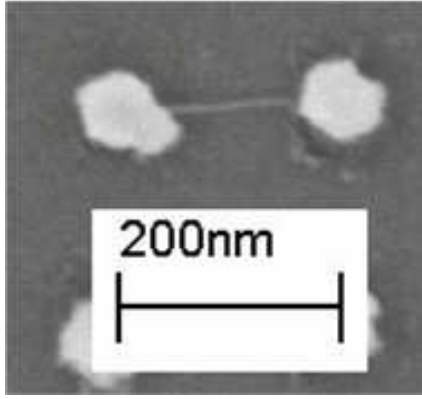


Fig. 4. Approximately 100nm long Single-Wall Carbon Nanotube Interconnect that Shows Good Alignment Between Two 30 nm Electron Beam Nickel Islands.

Using the process steps described previously, we have grown multiwall carbon nanotubes that can be used to develop MWCNT-based inductors. These MWCNT had physical size of 125nm in length and 25nm in diameter. We were able to grow multiwall carbon nanotubes that had 3 to 5 walls, which made our inductor design more attractive. Multiwall carbon nanotubes have attractive features to reduce the quantum resistance (contact resistance).

5. MEASUREMENTS

We have performed simulations on MWCNT-based stacked inductors for silicon and ceramic substrates using ADS software tool. To perform the comparison for different packaging, we have computed the substrate resistance and capacitance. The ceramic substrate resistance is 12 GΩ. The resistance of the silicon substrate was much lower, in the range of 100kΩ. The values of electrostatic capacitor C_E of each MWCNT were 15pF. We found coupling capacitances C_{c1} and C_{c2} to be 11pF. However, the coupling capacitance C_{c3} was approximately 6pF. These values were computed from Equation (8). We considered the distance between the walls to be 8nm. The substrate capacitances for silicon and ceramic substrates are 8pF, and 3pF, respectively. The quantum capacitance C_Q was computed as 0.2aF for each MWCNT.

Using Equation (3), the kinetic inductance L_k was computed as 2nH. The self-inductance for an MWCNT was computed with Equation (7). Their values are 0.028pH. Using the equation $L_M = k\sqrt{L_1 L_2}$, we computed mutual inductances. Their values ranged from 0.008pH to 0.015pH.

In general, the metal-based spiral inductors on Si substrate do not perform as well as the spiral inductors on organic substrate. We have used commercially available software tools [20] to obtain quality factor values. Using the software tool, the scattering parameters [s] were converted to admittance parameters [y] as shown in Equation (9).

$$[y] = [Y_0]([1] - [s])([1] + [s])^{-1} \quad \text{Eqn(9)}$$

$$\text{where } [Y_0] = \begin{bmatrix} Y_0 & 0 \\ 0 & Y_0 \end{bmatrix}.$$

The quality factor of the measured inductors was calculated by traditional method shown in Equation (10).

$$Q = -\frac{\text{Im}(y_{11})}{\text{Re}(y_{11})} \quad \text{Eqn(10)}$$

Table 1 shows quality factors for the MWCNT-based stacked inductor on silicon and ceramic substrates. The quality factor is impressive especially at 5 GHz, where IEEE standard 802.11a is used. The table shows that a Q factor of 3012 was achieved with the silicon substrate, and over 66,000 were possible for ceramic at 5GHz. We found that our self-resonant frequency of the inductor on a ceramic substrate was approximately 100Tera Hertz. This indicates that the MWCNT-based stacked inductors have extremely high bandwidth.

Table 1. Carbon Nanotube-Based Inductor Quality Factors for Silicon and Ceramic Substrates.

Freq (GHz)	(Q) Silicon	(Q) Ceramic
1	664	14232
2	1098	27659
3	1623	40125
4	2251	53042
5	3012	66278
10	3325	80125
20	3945	93658

The table indicates that MWCNT-based stacked inductors have very high Q factors. We have simulated self-resonant frequencies of these inductors and they were on the order of terahertz. This provided us with carbon nanotube inductors that can be used in a wide frequency spectrum. From our simulation results, we observed that ceramic substrates provided higher Q factors and inductance values. Figure 6 illustrates inductance values for silicon and ceramic substrates. From the figure, we can observe that the ceramic substrate provided approximately 195nH of inductance at 5GHz. These values seem very useful for today's applications of RF circuit design.

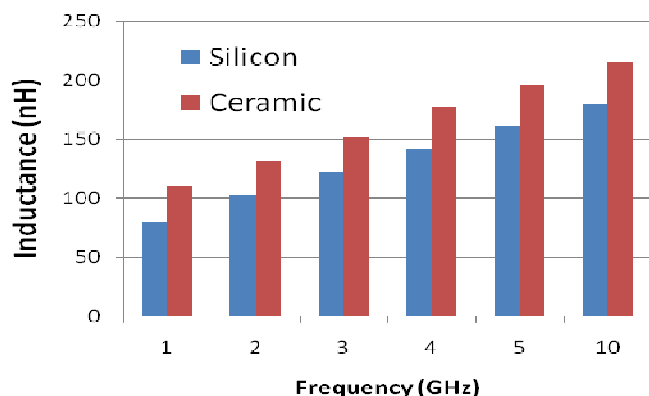


Fig. 6 Inductance Values for Carbon Nanotube Inductors on Silicon and Ceramic Substrates.

6. CONCLUSION

We proposed an alternative technique to synthesize an RF inductor by using carbon nanotubes. We have designed a stacked multiwall carbon nanotubes based RF inductors to provide high inductance and Q factors. Our simulation results indicate that high quality factors and inductance values can be achieved using carbon nanotubes. We believe that this work is highly promising for today's high-performance RF circuits and high-density packaging applications. We are currently fabricating CNT-based inductors to validate our model.

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