Advanced Modeling of Cure Shrinkage and Viscoelasticity for Warpage Prediction on Image Sensor Packaging

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Abstract

This paper presents a mechanical simulation of image sensor packages, with a particular emphasis on accurately modeling the cure shrinkage and viscoelasticity. Image sensors have a wide application in mobile phones, autonomous vehicles, and medical imaging. They are highly sensitive to warpage and misalignment, which can significantly degrade the image quality. The encapsulant acts as a primary barrier, protecting the sensor from external factors such as moisture and thermal mechanical stress. Accurately modeling and predicting the package warpage is crucial to ensure the optimal optical performance. Previous simulation studies on image sensor packaging often neglect cure shrinkage, leading to inaccurate warpage predictions. Cure shrinkage, a phenomenon where the material contracts during the curing process, can induce significant stress on the package and lead to warpage in addition to CTE-mismatch. This work developed a new simulation approach to incorporating cure shrinkage modeling, and it achieved a much higher degree of accuracy in predicting package warpage, as demonstrated by the close correlation between simulation results and actual warpage testing data. Furthermore, this paper studied the impact of using viscoelastic properties and compared time-dependent deformation with elastic solutions. Finally, this paper conducted comprehensive design of experiments (DOE) studies to evaluate the impact of different encapsulant materials on the susceptibility to glass cracking. The findings of this work are particularly useful for improved warpage prediction in simulation and better understanding of encapsulant properties, ultimately optimizing the package design and enhancing the reliability and longevity of electronic devices.

Key words

Cure shrinkage, Encapsulant, Image sensor packages, Simulation, Viscoelasticity, Warpage

I. Introduction

Image sensors are integral components in rapidly evolving technologies, including mobile phones, autonomous vehicles, and medical imaging devices. For example, sensors in autonomous vehicles provide accurate and reliable images for object detection, lane keeping, and other critical safety functions [1]. Because these applications demand increasingly higher levels of performance, they require high image fidelity. Image sensors are usually sensitive to warpage and misalignment, which will impact the accuracy of sensors in capturing the real-world scene [2]. Even minor distortion can significantly degrade image quality, impacting the overall effectiveness of the final product. One of the solutions is to shield it with encapsulant materials. Encapsulants can help minimize the warpage induced from coefficient of thermal expansion (CTE) mismatch and distribute stresses to reduce the risk of cracking or delamination. The choice of encapsulant material and its properties is critical in the design and manufacturing of image sensor packages. However, choosing the right encapsulant material involves time-consuming design of experiments (DOE) that can hinder the innovation cycle and prolong the time to market. Given the design and testing complexities, mechanical analysis using the finite element analysis (FEA) offers a cost-effective and faster alternative for evaluating various factors and their interactions with the package [3]. Accurate modeling and prediction of package warpage are crucial for both optimizing the optical performance of image sensors and ensuring their long-term functionality. A typical image sensor package is illustrated in Fig.1.

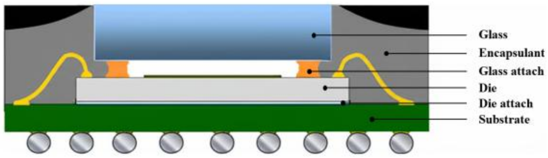


Figure 1. A typical image sensor package.

There have been a few studies on image sensor packaging, with a particular emphasis on understanding the impact of encapsulant materials. Kang et al. [4] reviewed and compared different image sensor packaging solutions e.g. ceramic-based, lead frame-based and laminate-based packages. Their paper highlighted the advantage of using a dam and liquid encapsulant material over traditional molding that results in fewer steps, cost reductions, and lower stress. Ying Jia et al. [5] used a thermal analysis to study the heat dissipation by encapsulant, and a mechanical analysis for warpage. Their simulation results showed a stress concentration at the center of the glass lid due to encapsulant where actual glass cracks occurred in experiment as well. However, the cure shrinkage was not considered in their simulation analysis. Cure shrinkage is a volumetric contraction during the curing process, and it introduces significant internal stress, which can lead to substantial warpage when combined with stresses from CTE mismatch. Jeong-Hyeon Baek et al. [6] measured the cure shrinkage of encapsulant in experiments and used FEA simulation to predict the warpage induced by cure shrinkage. They concluded that cure shrinkage must be considered in order for simulated warpage to match experimental results, but their structure was limited to a simple bi-material strip with encapsulant and aluminum, which is less applicable to semiconductor packaging or image sensors. Wei Keat Loh [7] performed both a mold flow analysis to understand the void formation risk on a flip chip package, and a mechanical analysis to study the impact of cure shrinkage and viscoelasticity on package warpage. They claimed that cure shrinkage played a significant role in determining the final package warpage. Moreover, the conversion rate of reaction also affected the glass transition temperature (Tg), and consequently, the evolving Tg caused more warpage than using a constant Tg in simulation. Other literature that studied interfacial strength, fatigue life prediction on encapsulant are [8] and [9]. In summary, for those who studied image sensor packaging, cure shrinkage has often been overlooked, and as a result, the warpage prediction is not accurate. Only a few simulation studies take cure shrinkage into consideration, but none of their applications are image sensor packaging.

This paper addresses the limitations of prevailing simulation on image sensors by presenting a novel simulation approach that incorporates the modeling of cure shrinkage. Cure shrinkage modeling is often not supported in the standard FEA packages, and so to be able to capture cure shrinkage effect, one typically needs to develop a user subroutine to characterize the conversion rate of reaction and their impact on modulus, etc. Hence, most studies just focus on thermal mechanical analysis, in which only the CTE-mismatch effect is captured. In current work, we developed a simplified approach for cure shrinkage modeling, which can be easily combined with traditional mechanical analysis and deployed in most of the standard FEA packages. With this approach, the impact of cure shrinkage on warpage and stress is captured. The improved accuracy of this approach is validated through a strong correlation between simulation results and experimental warpage data. Furthermore, this work investigates the influence of the encapsulant's viscoelastic properties on package deformation, comparing time-dependent behavior with elastic solutions. Finally, a comprehensive design of experiments (DOE) is conducted in simulation to evaluate the effects of varying encapsulant material properties on the susceptibility of the package to glass cracking. The results of this study provide valuable insights for enhancing the accuracy of warpage predictions in simulations, particularly due to cure shrinkage, and improving the understanding of encapsulant material behavior, ultimately facilitating the design of more reliable and robust image sensor packages.

II. Methods

A. FEA model

The FEA simulations were conducted using ABAQUS. The computer-aided design (CAD) was created based on the specification provided by the test vehicle assembly. A python-based script was developed to streamline and automate the CAD process. By doing so, it can save time for repetitive DOE studies, reduce manual errors in sketching geometry, and focus more on post-processing analysis. An example of the geometry created by using python-script is shown in Fig. 2. With the tie constraints, parts do not have to be merged into assembly. It allows different regions to bond together at interface even though meshes created on the surfaces of different regions may be dissimilar. Each part is meshed with its own size control for computational efficiency, and all is meshed using the second-order 20-noded brick element (C3D20R) for improved accuracy. An example of the mesh is shown in Fig. 3. The corner nodes are simply supported to remove the rigid body motion. A uniform thermal loading is applied to the entire package. The reference temperature is chosen at 150 ℃ for initial thermal condition. Material properties used in the FEA model are listed in Table. 1. The key aspects of cure shrinkage and viscoelasticity are described in the following sections.

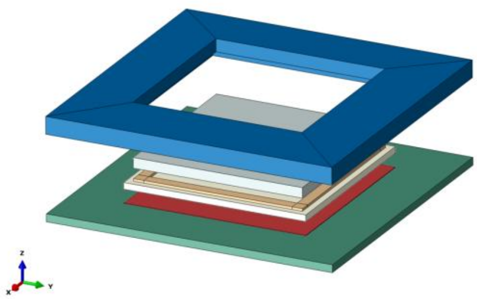


Figure 2. CAD geometry of the image sensor package.

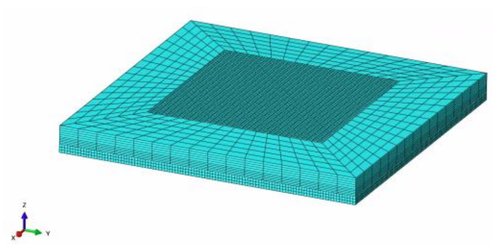


Figure 3. Mesh of the image sensor package.

Table 1. Material properties used in the FEA model.

Name	Properties
Glass	E=64GPa, v=0.2, CTE=3.25ppm/C
Glass attach	E=5.97GPa, v=0.3, CTE1=39ppm/C, CTE2=128ppm/C, Tg=66C
Die	E=169GPa, v=0.26, CTE=2.62ppm/C
Die attach	E=1.54GPa@-65C, 159MPa@25C, 13MPa@150C, v=0.3, CTE1=125ppm/C, CTE2=221ppm/C, Tg=-15C
Encapsulant	E=23.7GPa@-65C, 19.6GPa@25C, 63MPa@150C, v=0.3, CTE1=14ppm/C, CTE2=44ppm/C, Tg=48C
Substrate	E=27.3GPa@20C, 25.7GPa@100C, 21.6GPa@150C, v=0.19, CTE: XY=14.5/5.5, Z=32.5/150, Tg=190C

B. Cure shrinkage

To accurately model cure shrinkage, one needs to know how the degree of cure evolves over time and temperature. This is described by cure kinetics, which is usually determined from Differential Scanning Calorimetry (DSC) data. A commonly used model is the so-called Kamal model [10], which directly relates the rate of cure to the current degree of cure and temperature as follows

$$\frac{d\alpha}{dt} = (k_1 + k_2\alpha^m)(1-\alpha)^n \tag{1}$$

where α is the degree of cure that goes from 0 to 1, k_1 and k_2 are temperature-dependent rate constants, and m and n are empirical exponents. Determining k_1 and k_2 for the Kamal model can be quite challenging, as it involves conducting isothermal DSC experiments, analyzing the heat flow data to obtain the rate and degree of cure, and then using regression to fit the Kamal model. Moreover, the accuracy of the determined constants heavily relies on the quality of the isothermal DSC data, which can be easily polluted by noise and improper temperature control. Given the complexities in Kamal model calibration, the volumetric strain method is a significantly simplified and practical approach to include cure shrinkage in FEA if the transient effect is not a concern.

In the volumetric strain method, a relationship between the volumetric strain and the degree of cure is assumed to exist, the simplest and most common assumption is a linear relationship, namely,

$$\varepsilon_{vol} = -\beta\alpha \tag{2}$$

where ε_{vol} is the volumetric shrinkage strain, and β is the total volumetric shrinkage at full cure ($\alpha = 1$). To measure the volumetric shrinkage, one would typically measure the density of the resin before and after curing, then calculate the volumetric shrinkage. A pycnometer is used to measure the density of the liquid resin, while a densometer or similar device is used to measure the density of the solid cured resin. The difference in density between the two states is then used to calculate the volumetric change, which is the cure shrinkage.

The relation for stress and strain with thermal effects is

$$\sigma = D(\varepsilon - \alpha_T \Delta T) \tag{3}$$

where σ is the stress tensor, and D is the material stiffness matrix, ε is the total strain, α_T is the CTE with a subscript "T" added to distinguish it from the rate of cure, and ΔT is the change in temperature. Now, with additional cure-induced strain, the constitutive relation becomes

$$\sigma = D(\varepsilon - \alpha_T \Delta T - \beta\alpha) \tag{4}$$

In FEA software, the cure-induced volumetric strain is often introduced through a user-defined material subroutine or a predefined material model. Here in this paper, we use a 'predefined field variable' to add the cure shrinkage strain. In ABAQUS, the field variables are used to represent spatially varying quantities like temperature or material properties. They are crucial for non-linear analysis and for simulating special scenarios. In this case, we are particularly interested in the volumetric contraction because of cure shrinkage. Field variables can be defined using a variety of methods, including specifying values directly, using analytical or discrete fields, or defining them in user subroutines. Unfortunately, the predefined field variables are not supported in ABAQUS CAE per user manual [11]. Hence, one must either add the field variable to the input files or edit the keyword in the model itself manually, as shown in Fig. 4. It is noted that the field variable here is defined as a linear expansion coefficient, hence one must convert the volumetric strain to an equivalent linear strain.

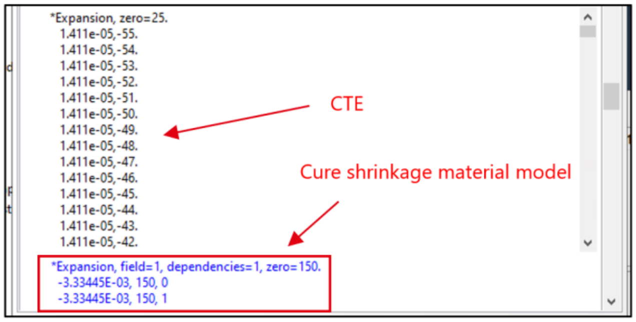


Figure 4. The volumetric strain method of cure shrinkage using the predefined field variable in ABAQUS. Here assumes a 1% volumetric shrinkage in encapsulant.

C. *Viscoelasticity*

Unlike purely elastic materials that deform instantaneously under loading and return to undeformed shape upon unloading, viscoelastic materials display time-dependent deformation, e.g. stress relaxation, as illustrated in Fig. 5.

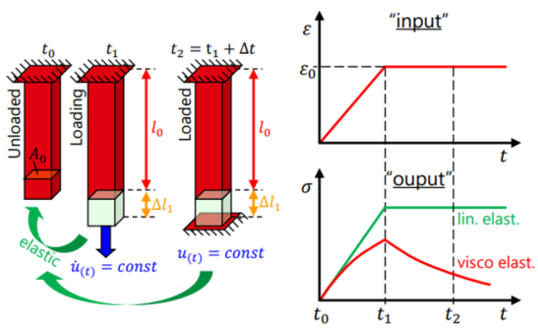


Figure 5. Linear elastic material vs viscoelastic material.

Viscoelastic behavior can be mathematically described using various models. A common and effective approach is the Prony series [12], which represents the time-dependent relaxation modulus, $G(t)$, as a sum of exponential terms:

$$G(t) = G_0\left(1 - \sum_{i=1}^{N} g_i\left(1 - e^{-\frac{t}{\tau_i}}\right)\right) \qquad (5)$$

where G_0 is the instantaneous modulus, g_i are the normalized relaxation moduli, and τ_i are the relaxation time in decay. Using the Fourier transform, the expression for the time-dependent shear modulus can be written in the frequency (ω) domain as follows:

$$G(\omega) = G_s(\omega) + iG_l(\omega) \qquad (6)$$

where the storage modulus $G_s(\omega)$ and the loss modulus $G_l(\omega)$ are

$$G_s(\omega) = G_0\left[1 - \sum_{i=1}^{N} g_i\right] + G_0\sum_{i=1}^{N}\frac{g_i{\tau_i}^2\omega^2}{1+{\tau_i}^2\omega^2} \qquad (7)$$

$$G_l(\omega) = G_0\sum_{i=1}^{N}\frac{g_i\tau_i\omega}{1+{\tau_i}^2\omega^2} \qquad (8)$$

To use the Prony series in FEA simulations, the parameters need to be determined experimentally. This process typically involves performing a Dynamic Mechanical Analysis (DMA) test at relevant temperatures and frequencies, and fitting the experimental data to the Prony series equation. The DMA test details are as follows. A temperature sweep is performed first to make sure all important temperatures and phase changes are captured. A small strain amplitude 0.1% is used to make sure material's response is in the linear viscoelastic range. The frequency sweep is performed from 1 Hz to 25 Hz. The master curve is constructed by horizontally shifting the frequency sweep data at each temperature to align with the reference temperature at 25°C, using the Williams-Landel-Ferry (WLF) model. This approach ensures that the viscoelastic behavior across a wide range of timescales is accurately captured and reflects the material's response. The fitting can be done using software or optimization algorithms to determine the best fit values for the Prony parameters. Fig. 6 shows the viscoelastic characterization for encapsulant material. The accuracy of the viscoelastic model depends on the quality of the experimental data and the number of Prony terms used. More terms generally provide a better fit but increase the complexity of the model.

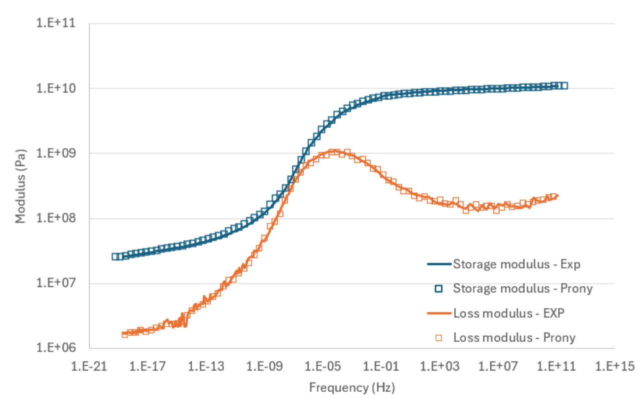


Figure 6. The viscoelastic characterization of encapsulant material – experiment vs Prony series.

D. *Test vehicle assembly*

A test vehicle was designed to replicate the essential structural elements of an actual image sensor package, allowing for experimental testing of the mechanical behavior, as well as to validate the FEA simulation. Fig. 7 shows the test vehicle before singulation, where multiple image sensor packages are built onto one substrate for efficiency. Fig. 8 shows the singular image sensor package after singulation from different views. The assembly process illustrated in Fig. 9 comprises the key steps in die attachment, glass attachment, encapsulation, and singulation. Optional steps such as reflow, or post solder ball attachment are not included here. Dimensions of the test vehicle are in Table.2. Dimensions such as die size and encapsulant thickness can directly affect the stress distribution, warpage and reliability performance. Hence, dimensions are chosen carefully in the test vehicle design to ensure representative and normal package behavior. Current work focuses on the 7x7mm die size only. Study on die size variations is beyond the scope of current work. Materials and processes are selected to match those used in the actual image sensor package design.

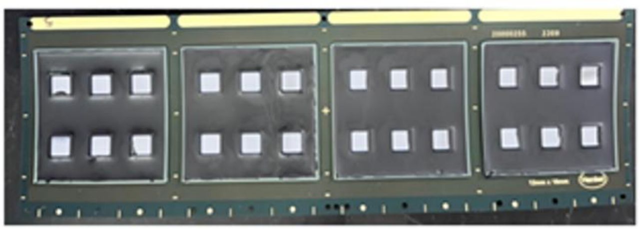
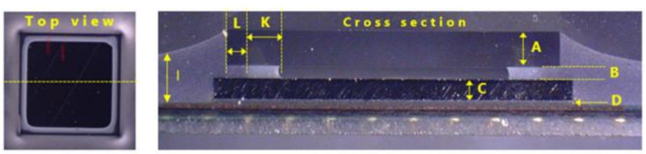


Figure 7. Top view of the test vehicle before singulation



(a) (b)

Figure 8. The singular image sensor package from (a) top view and (b) cross-section view

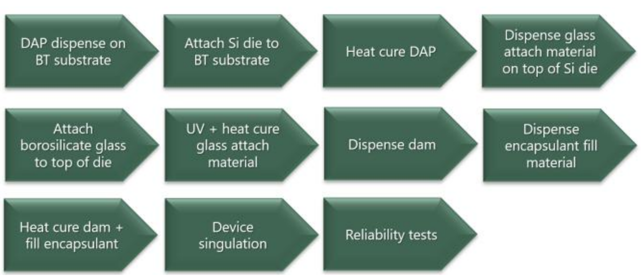


Figure 9. The process flow for the image sensor test vehicle fabrication.

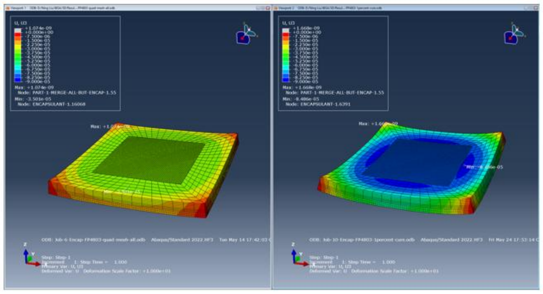
Table 2. The dimensions of the image sensor test vehicle

Symbols	Description	Dimensions (um)
A	Glass lid thickness	400
B	Glass attach thickness	150
C	Si wafer thickness	250
D	Die attach thickness	30
I	Encapsulant thickness	600
K	Glass attach width	375
L	Glass edge to glass attach	170

III. Results and discussion

This section presents the findings from the FEA simulations and compares them with experimental results to validate the accuracy of the proposed methodology. The impact of incorporating cure shrinkage into the simulation using the volumetric strain method is evaluated, followed by a comprehensive DOE study to assess the influence of encapsulant material properties on package warpage and stress.

The simulations in this work rely on assumptions about the cure shrinkage. Hence, experiment validation is necessary and crucial to assess accuracy and increase confidence in simulation results, allowing this FEA model to be used reliably for predicting package performance. The simulation is first carried out on the singular image sensor packages. Warpage predictions with and without considering cure shrinkage are in Fig. 10. Including viscoelastic properties does not affect the warpage magnitude significantly but reveals the warpage relaxation over time. For simplicity, only the elastic properties are included in the simulation for cure shrinkage in this paper. The simulation with viscoelastic effect and its impact on glass stress is discussed in the next section. It is seen that warpage prediction is much higher with cure shrinkage than without. Warpage measurements are obtained from the test vehicle using the Keyence 3D optical imaging to validate the simulation results in Fig. 11. Experiments and simulation results are compared at room temperature in Fig. 12. Error reduces from 42% to 16%, showing that the warpage prediction is much closer to the experiment with cure shrinkage effect included. This proves that the proposed cure shrinkage modeling is reliable, as it accurately captures the influence on warpage. It also highlights the significance of including cure shrinkage in simulation because it leads to a more accurate prediction. The limitation of this method, however, is that it neglects the time and temperature dependency, which can affect the cure induced warpage. Moreover, modulus changes as a function of the degree of cure. But the simulation of curing process is not the major focus of this work, thus the material is assumed to be fully cured prior to the thermal mechanical simulation, and the material stiffness is considered constant in the curing process.



(a) (b)

Figure 10. The warpage prediction from simulation (a) without cure shrinkage and (b) with cure shrinkage.

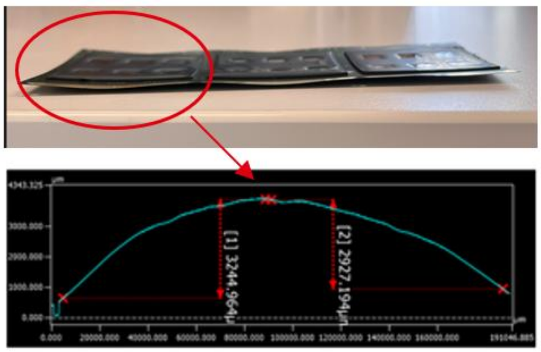


Figure 11. The warpage measurement on the back side of the test vehicle (substrate) using Keyence 3D optical imaging.

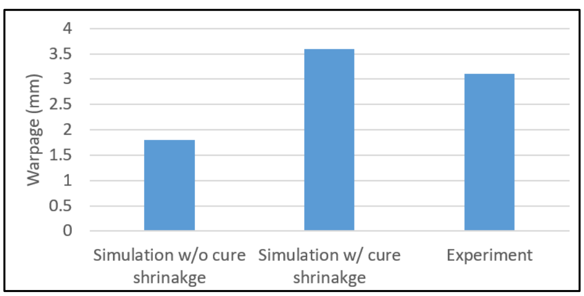


Figure 12. The simulation vs experiment results on warpage. With cure shrinkage, the simulation result is much closer to experiment.

Encapsulant materials used in image sensor packaging exhibit viscoelastic behavior. Accurate modeling of viscoelasticity in encapsulant is crucial for accurate warpage and stress prediction over time. Fig. 13 shows the temperature profile of a thermal cycle test. One cycle of the thermal cycle test is simulated here to evaluate the time-dependent effect. Fig.14 shows stress concentration at the glass edge. Glass cracking is a critical failure in image sensor packages. Stress concentration, often exacerbated by CTE mismatch between encapsulant and the glass, can lead to cracks, compromising optical performance and device integrity. Fig.15 shows the influence of encapsulant's viscoelasticity on glass stress. Using viscoelastic model predicts slightly higher stress than using elastic model, so it presents a higher risk for glass crack. Furthermore, there is residual stress in glass at the end of the cycle. This is due to the viscous effect of encapsulant's viscoelasticity, which leads to a delayed response and a retention of stress on encapsulant even after deformation stops. The residual stress on glass will carry over to the next cycle and build up significant stress eventually leading to failure. Contrarily, elastic solutions do not capture residual stress, resulting in inaccurate predictions. Therefore, viscoelastic modeling of encapsulant is necessary to predict a time-dependent deformation, and also critical to achieve a more accurate simulation result, especially in capturing the influence of encapsulant on glass crack.

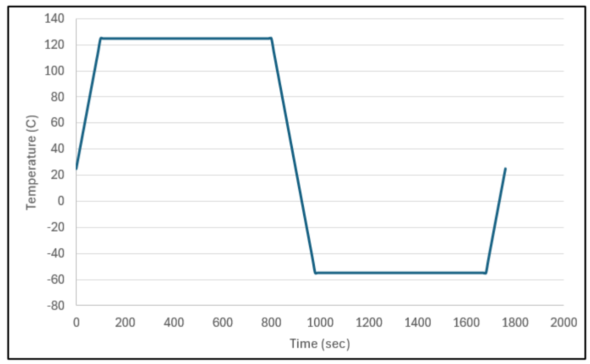


Figure 13. Temperature profile of a thermal cycle test.

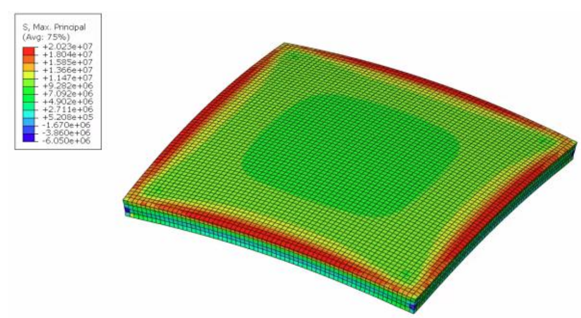


Figure 14. Stress concentration at glass edge.

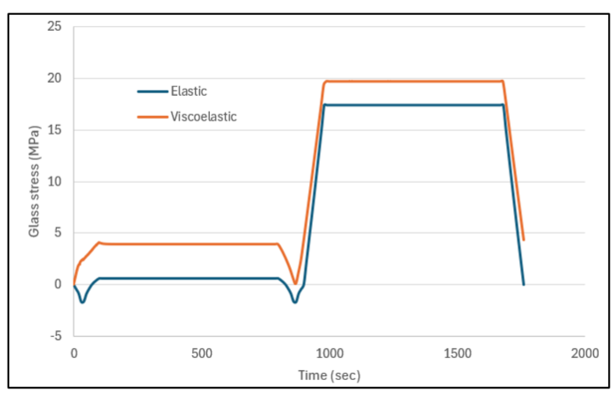


Figure 15. Influence of the encapsulant's viscoelasticity on glass stress in comparison to elastic solution.

DOE studies are essential to systematically investigate how encapsulant material properties influence package reliability, particularly concerning glass cracking. Fig. 16 shows the DOE study on modulus. Encapsulant with higher modulus causes higher stress on glass, which increases the risk of glass crack. But warpage is not significantly affected by modulus. Fig. 17 shows the DOE study on CTE. Encapsulant with higher CTE causes significantly higher warpage, but only minor increase on glass stress. It is worth mentioning that the warpage shape is flipped from one side to the other as CTE increases. This shows that a lower CTE for the encapsulant is not always optimal. Instead, there exists a specific CTE value that minimizes the mismatch with other package components, thereby reducing warpage. Fig. 18 shows the DOE study on cure shrinkage. Trend is similar to that of CTE. Encapsulant with higher cure shrinkage can cause both more warpage and more glass stress.

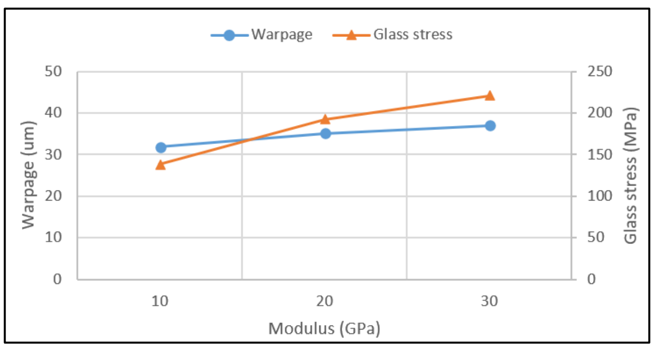


Figure 16. Encapsulant DOE #1 – influence of modulus on package warpage and glass stress.

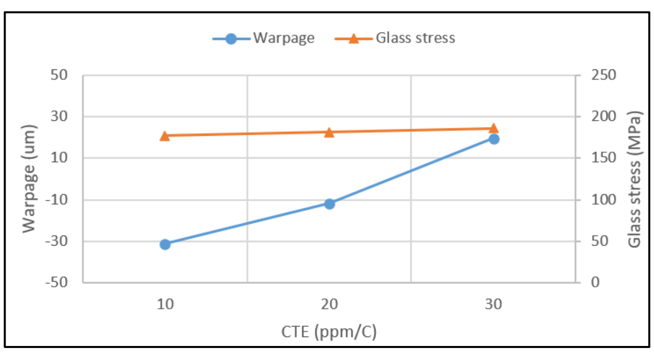


Figure 17. Encapsulant DOE #2 – influence of CTE on package warpage and glass stress.

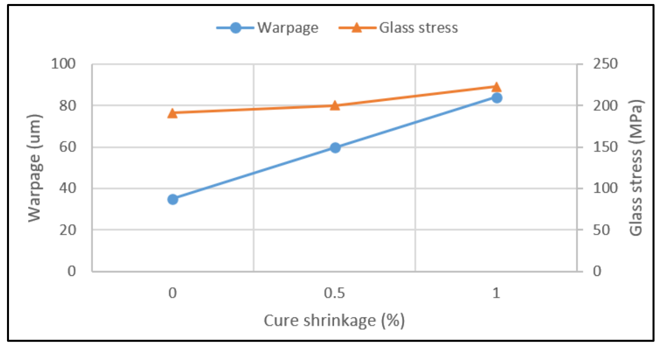
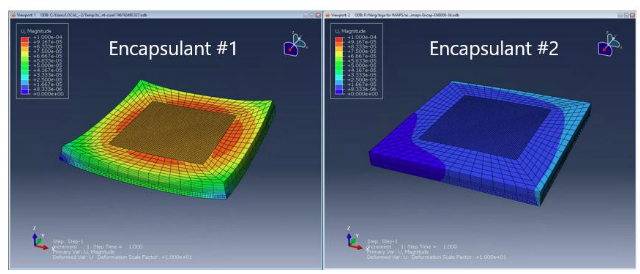
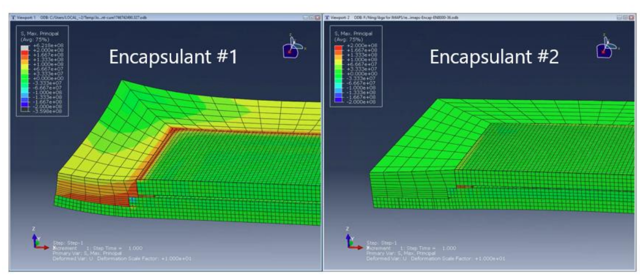


Figure 18. Encapsulant DOE #3 – influence of cure shrinkage on package warpage and glass stress.

Based on the insights gained from the DOE studies, a new encapsulant material with optimized properties was designed from baseline. Simulations using this new candidate encapsulant predict a significant reduction in both package warpage and glass stress, as in Fig. 19 and Fig. 20. This demonstrates the effectiveness of the DOE-driven approach in optimizing encapsulant properties and ultimately enhancing the reliability performance of the image sensor packages. The next step involves collaborating with material scientists to fine-tune the formulation of this new encapsulant candidate, followed by experimental verification of its performance.



(a) Warpage



(b) Glass stress

Figure 19. Simulation results for encapsulant #1 (baseline) and #2 (new) in terms of warpage and glass stress.

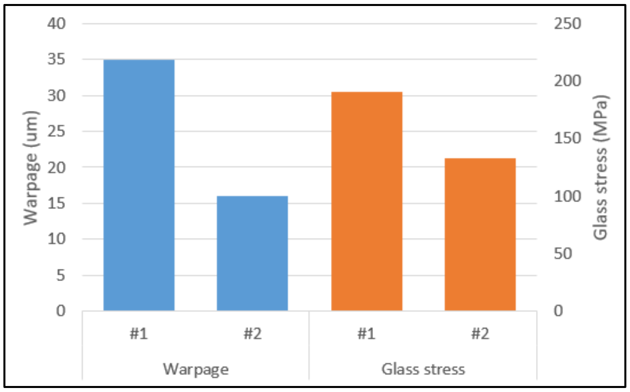


Figure 20. The encapsulant #2 shows clear reduction in both package warpage and glass stress compared to #1.

IV. Conclusion

This paper addresses the critical issue of package warpage in image sensors, which can significantly degrade optical performance and long-term reliability. The work highlights the importance of accurately modeling the cure shrinkage effect and the viscoelastic behavior of encapsulant materials, which are often neglected in traditional simulation studies. The proposed FEA method, incorporating cure shrinkage, demonstrates a significant improvement in accuracy of warpage prediction compared to simulations that do not account for this phenomenon. Accurate modeling of viscoelasticity is also crucial for capturing the residual glass stress caused by the viscous effect, a phenomenon that elastic solutions cannot predict. Furthermore, the comprehensive DOE study provides valuable insights into the influence of encapsulant material properties, namely modulus, CTE, and cure shrinkage on package warpage and glass stress. These findings contribute to a better understanding of the factors governing image sensor package reliability and provide a foundation for optimizing material selection and package design.

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